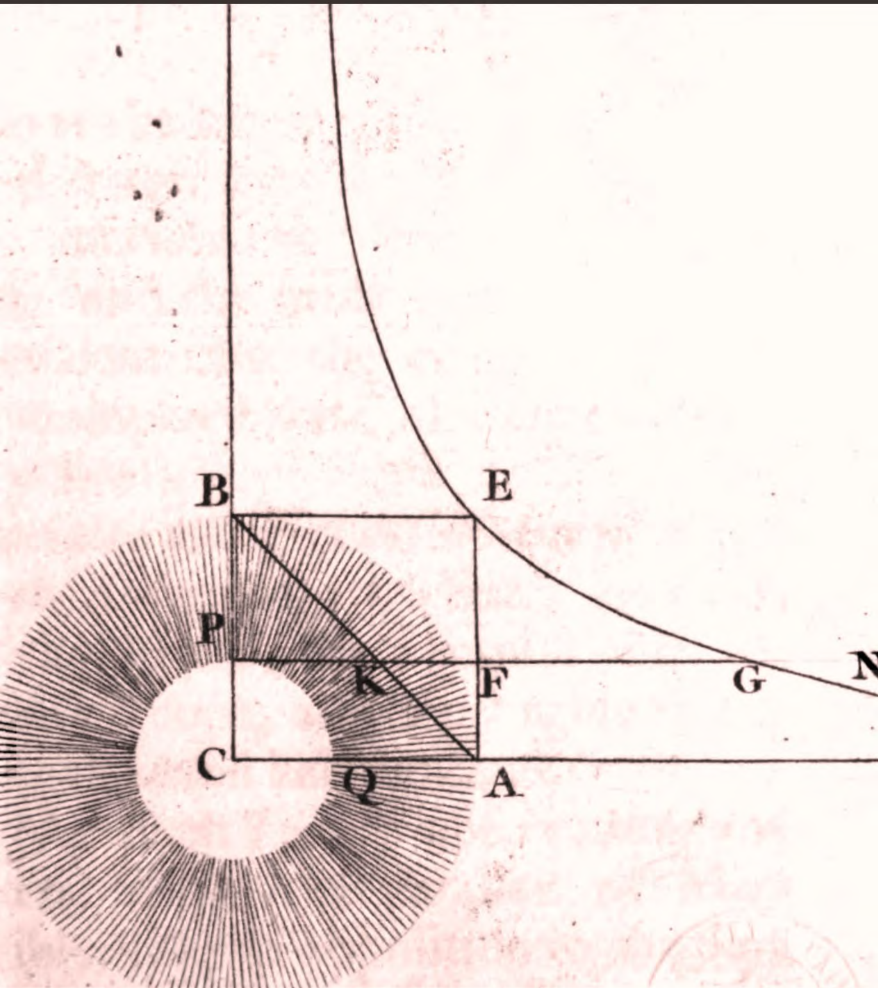


A treatise of the system of the world

Isaac Newton, Andrew Motte


Ex Libris
RIEU

A

TREATISE

OF THE

SYSTEM

OF THE

WORLD.

BY

Sir *ISAAC NEWTON.*

Tranſlated into ENGLISH.

LONDON:

Printed for F. FAYRAM at the South Entrance under the *Royal Exchange.*
MDCCXXVIII.

PREFACE.

IN the beginning of the third book of the Mathematical Principles of Natural Philosophy of the late illustrious Sir ISAAC NEWTON, *he acquaints us, " that he had once " composed that book in a popular " method that it might be gene- " rally read; but that afterwards " to avoid the disputes which might " arise, if those who were unac- " quainted with the principles laid " down in the preceding books, and " full of the prejudices which many " years had made natural to them, " should take it in hand, he put " the substance of that book into " propositions in the mathematical " way, that it might be read by " those only who had studied the " principles beforehand." The pre- judices that great Man apprehend-*

 ed,

ed, when the principles whereby he explained the appearances of Nature were new, are now ſo well diſſipated by the univerſal applauſe and approbation they have been juſtly attended with, that his reaſon for ſuppreſſing that performance being entirely ceaſed, we think the ſame may be very acceptable to the World. And to render it the more uſeful to our countrymen, we chuſe to give it in a careful tranſlation rather than in the original Latin.

That this work as it is here publiſhed is genuine, will ſo clearly appear by the intrinſic marks it bears, that it will be but loſing words and the Reader's time to take pains in giving him any other ſatisfaction. It will be more convenient to clear the way for thoſe who are leſs acquainted with the ſubject, by a ſhort account of Aſtronomy, and the ſtate it was in when the Author wrote; by which the curious

curious discoveries therein contained will be the better understood.

By the imperfection of our sense it comes to pass, that the appearances of the heavens and heavenly bodies are entirely different from what they really are in themselves. The Sun, the Moon, the other Planets, and the fixed Stars, appear to the eye only like small circles or like points of light, at equal distances from us; sticking to the top of a blue hemispherical vault, not very far from the Earth, which it seems to touch on every side near the horizon. And this blue vault turning upon its axis, these bodies being carried round with it, rise and set alternately, and so present themselves in order to the several parts of the Earth.

But our reason assisted by geometry, opens to us a very different scene. It shews us an immense ocean of celestial space spreading

it ſelf on all hands from our Earth without poſſibility that any limits ſhould be ſet to it. That the ſeveral heavenly bodies that ſeem to our eye ſo ſmall, and equally diſtant from us, are in reality of prodigious magnitudes, and placed at vaſt and very different intervals from the Earth and each other, in that boundleſs abyſs. Theſe things were long ago known and conſented to by all; but the exact form in which theſe bodies and the Earth are ſituate with reſpect to each other in that immenſity of ſpace, was not well ſettled till about ſixty years ago; at which time Aſtronomers generally agreed in the following Syſtem. That the Sun, though it ſeems to move, is indeed fixed and quieſcent in a certain given part of the celeſtial regions. That at a great diſtance from thence that Planet we call Mercury goes round him in an orbit either circular or very nearly ſo, in the centre of which the Sun is.

That

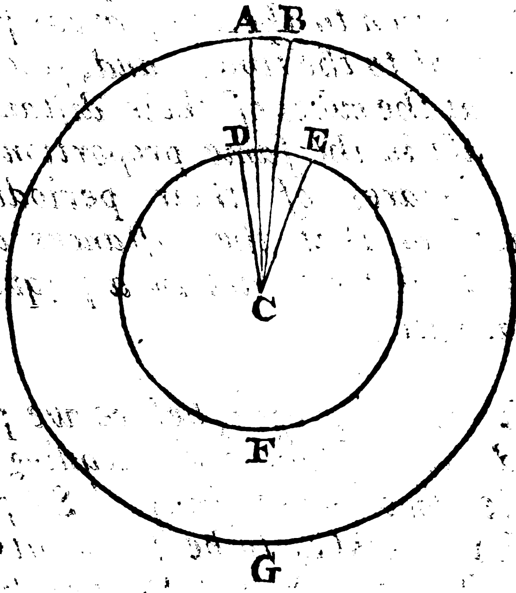
That at a yet greater distance the Planet Venus revolves about the Sun in like manner. That next to Venus the Earth goes round the Sun, turning at the same time upon its axis, and attended by the Moon; which revolves about the Earth as the Earth does about the Sun. That the other Planets, Mars, Jupiter, and Saturn, move also round the Sun in like orbits with the others; the two last being observed to have Satellites or Moons revolving about them, as our Earth has its Moon. All this is represented in the common figures of the Solar or Copernican *System.*

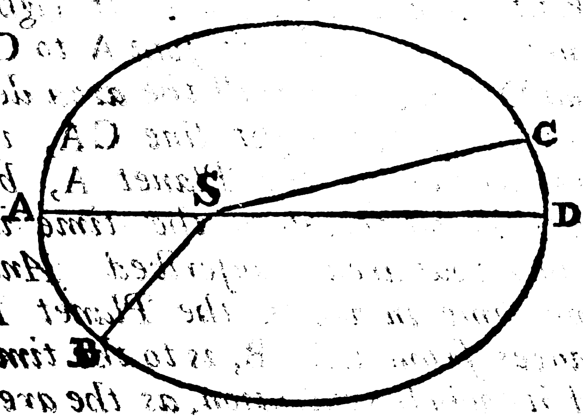
But Astronomers did not stop here. They not only found the order in which the Planets move, and the situation of their orbits; but the sagacious Kepler *made also the following great discoveries concerning the laws of their motions. The first, that all the Planets, in*

moving round the Sun, deſcribe, by radii drawn to the Sun, areas proportional to the times: and, ſecondly, that the cubes of their diſtances increaſe in the ſame proportion as the ſquares of their periodical times, or that the diſtances and periodical times are in a ſeſquialteral ratio.

It may be proper, before we proceed, to explain the meaning of theſe two more fully. Suppoſe DEF, ABG, *to be the orbits of two Planets* A *and* D, *revolving about the Sun at* C; *now if right lines be conceived to join* A *to* C, *and* D *to* C, *then will the area deſcribed by the right line* CA, *in the motion of the Planet* A, *be ever proportional to the time in which that area is deſcribed. And the time in which the Planet* A *moves from* A *to* B, *is to the time of its whole revolution, as the area* ACB, *is to the whole area* ABGA.

Likewiſe


A
B
D
E
C
F
G


C
A
S
D
B

Likewiſe the area deſcribed by the right line CD, *in the motion of the Planet* D, *is ever as the time in which that area is deſcribed. And the time in which the Planet* D *moves from* D *to* E, *is to the time of its whole revolution, as the area* DCE, *is to the whole area* DEFD. *So alſo we may ſuppoſe a Planet moving in an orbit* ABDC, *of an elliptical figure, in whoſe focus at* S *the Sun is. Here the Sun will be nearer to ſome parts of the orbit than to others, as to* A *and* B *more than to* C *and* D. *In this caſe, if areas proportional to the times are deſcribed by radii drawn to the Sun at* S, *by how much the diſtance* CS *or* DS *is greater than* AS *or* BS, *by ſo much will the arc* CD *deſcribed in any given portion of time, be leſs than the arc* AB *deſcribed in the ſame time. And this, as was ſaid, is what we mean by this great law of proportionality*

of

of areas and times to which the motions of the Planets are subject.

In the next place we may explain a little the nature of the other law of the planets motions. The meaning is this, that if we take the cubes of the distances of any two Planets, as Venus, and Mars (which are known by astronomical calculation and observation,) and the squares of the periodical times of the same Planets, (known by the same methods,) these four numbers will be in proportion: that is, the square of the time in which Venus moves in the orbit DEF, *is to the square of the time in which Mars moves in the orbit* ABG, *as the cube of the distance* CD, *is to the cube of the distance* CA. *And quantities in that proportion are said to have a sesquialteral ratio to each other.*

This

This was the state of the astronomy of the Planets, when Sir Isaac Newton *wrote. As to the Comets, they were generally thought to move uniformly in right lines through the celestial spaces, and not to describe any orbit recurring into it self, or at all incurvated; and it was a prevailing opinion, that at the time those bodies were visible, their motions were performed in the regions beyond Saturn.*

There remained one thing farther to be known, which was, the cause which retains the Planets in those regular motions which they are found to revolve with. For since projected bodies, impelled by a single force, move by their own nature uniformly along the right lines in which the forces that impel them are directed, without any possibility of deviating from those rectilinear paths; it follows that bodies, like the Planets, that move in curve

lines,

lines, must be acted on by more forces than one, or by some compound force, drawing them out of the right lines they naturally tend to describe. The question then was, of what nature that force is which is diffused through so vast an extent as that of the Planetary System, and which seems to be simple and uniform, by its uniform effects above-mentioned, of the proportionality of the areas and times; and the sesquialteral ratio of the distances and periods, said above to be collected from the observations.

To this end the Author did not frame hypotheses as other philosophers used to do, but set himself to examine the phænomena themselves, by mathematical reasoning. And first of all, he found that bodies describing areas proportional to the times by radii drawn to any given point are impelled by two forces; by one of which they continually endeavour to fly off in the tangents

tangents of the orbits they describe, and by the other, which continually impells them to the given point, they are with-held from doing so, and retained in those curvilinear paths.

Since then the Planets by radii drawn to the Sun describe areas proportional to the times, it follows that the compound force which keeps them in their regular motions according to that law, is compounded of two forces; one of which impells them according to the tangents of their orbits, and the other impells them towards the Sun; the first of which we may call the Projectile, the other the Centripetal Force.

Now by this discovery we have made a great advance, having found, First, that this phænomenon of the motions of the Planets may be accounted for by two such forces; and, Secondly, that the centripetal force is directed to one fixed and given

given place, the Sun's centre; and not to any uncertain and wandring places; which would have made our future speculations very intricate, if it had proved so. What the cause is of this force, we do not yet pretend to determine; our business is, since such a force is found to exist, to search into the properties and proportions of that force, before we think of enquiring into the cause of it.

The next question is then what proportion two centripetal forces actually bear to one another in two different Planets. For forces being mensurable quantities, are capable of being compared together as to proportion; which proportion may be expressed by numbers; in the same manner as the proportions of any two lines, weights, &c. And since these two forces actually exist, and operate upon the Planets, they must of course bear some certain proportion to each other, if that could

could be found out. They must be either equal in all the Planets, or greater in the nearest, and less in the more remote, or vice versa.

The investigation of these proportions is a mathematical problem; which the Author solves by finding that if several bodies revolve about a centre in such orbits as the Planets, and the times of their periods be to each other in a sesquialteral ratio, their centripetal forces will be reciprocally as the squares of the distances from the centre. Whence it follows that since the Planets move about the Sun in periods having that ratio to each other, their centripetal forces towards the Sun are reciprocally as the squares of their distances from the Sun.

There is therefore a virtue diffused throughout the whole Planetary System by which the Planets are obliged to tend on all sides towards

wards the Sun placed in the common center or focus of their orbits; and whose effect is always in the reciprocal duplicate ratio of the distances from the Sun.

This is a discovery for which we are beholden to the penetrating genius of our great Author. Our next enquiry must be, whence this virtue is propagated. For though we know such a virtue exists, we are not yet certain whence it is produced.

To set us something more forward in this search, the Author has discovered and demonstrated by mathematical reasoning, that if a globe is composed of particles to each of which there tend centripetal forces that are in the reciprocal duplicate ratio of the distances from the particles, the centripetal forces tending to the whole globe will be also in the same ratio. Therefore, since there are centripetal forces tending to the whole body of the Sun which are in that reciprocal duplicate ratio of the

distances, it follows that each single particle of the Sun's body has such centripetal forces tending to it acting in the very same ratio. So that we see the deflection of the Planets from the rectilinear paths into the curve orbits they describe, is occasioned by a certain property of the parts of which the Sun is composed, which by some hidden virtue, either by attractions propagated from those parts or by impulses directed to them, oblige the Planets to tend towards the Sun's body perpetually, with forces that are in the reciprocal duplicate ratio of the distances from the Sun. This virtue we will name an attractive virtue; though the effect thereof is more probably caused by an impulse from without the attracted body, rather than by any thing emitted from that which is said to attract. But since the names of things are arbitrary, and after this caution premised, this name cannot mislead us, we use it for the convenience of brevity.

But

But by the laws of motion received by all philoſophers, it is certain that if the particles of the Sun attract the particles of the Planets, theſe latter muſt alſo equally and mutually attract the former. And if ſo, it follows that the particles of the Sun and of the Planets are, in reſpect of this attractive virtue, entirely ſimilar. By Planets, I here mean the primary. But ſince there can be no pretence of reaſon to imagine that the ſecondary Planets are in this reſpect different from the primary, it follows that all the bodies in the Planetary Syſtem are endued with this virtue, of attracting others in the reciprocal duplicate ratio of their diſtances from the attracted body. And if every ſingle particle of each body be ſeparately endued, as was ſhewn above, with this virtue, it follows that where a number of thoſe particles are united in one body, the entire force of that whole body will be cæteris paribus *as the number of thoſe particles; that is, as its quantity*

of matter. And so the attractive virtue of all bodies will be cæteris paribus *as their quantity of matter.*

Therefore all the Planets may be considered as centers and sources of this attractive force which spreads it self about them on all sides, attracting towards them from all quarters, all bodies that are not hindered by a more forcible attraction to some other body. This is farther confirmed by the revolution of secondary Planets about some of the primary, (as about the Earth, Jupiter and Saturn) according to the same laws, which the primary observe about the Sun, as to the proportionality of times and area's, and the sesquialteral ratio of the distances and periods: and the same thing appears by the sensible effects of the Moon upon the Earth. All which demonstrate that an attractive virtue is propagated from those bodies, of the very same kind with that which is propagated from the Sun.

And ſince, as was ſhewn above, the centripetal forces of the Sun and primary Planets are in a given ratio to the periods and diſtances of the primary Planets about the Sun, and of the ſecondary about the primary, it follows that we may mathematically inveſtigate from the phænomena the proportions which the centripetal forces of thoſe celeſtial bodies ſeverally bear to each other; and from thence judge of the effects they are ſeverally like to produce in the Syſtem of the World. And the doing this is the ſubject of the firſt part of this tract.

The ſum therefore of theſe new inventions is contained in this great and univerſal truth, and its corollaries, That all bodies through the whole ſolar Syſtem [and therefore throughout the univerſe, as we have all the reaſon that can be to ſuppoſe] attract, or (which is the term uſed moſt generally) gravitate towards each other with forces that are as their quantities of matter directly, and as the ſquares of their diſtances from each other reciprocally.

On

On this great foundation the Author builds two important discoveries, entirely new; the first relating to the Moon and the tides; the second relating to the motions of the Comets, and the figure of their orbits.

Astronomers have always found the Moon's motion to be subject to so many and various irregularities that they could never bring their tables to answer to it, with any thing near the exactness they could do in the other Planets. And besides, the Phænomena they observed in her motion were in themselves very odd and surprizing, there being no example of the like in any of the other Planets. But from that great law of nature, abovementioned, the true source of those irregularities has been most clearly laid open by this great writer. And from the same principles that wonderful Phænomenon of the ebbing and flowing of the sea, is here also most plainly deduced.

The Astronomy of Comets was entirely unknown till the time this Tract was written. The figure of the paths they

they described, the regions through which they passed when they were visible to us, and those to which they afterwards went, were quite mistaken by all the Astronomers. But they are now so clearly made known to us by the Author's means, that nothing is wanting to compleat that Astronomy but length of time and opportunities of observations. The construction of a Comet's trajectory with which this Treatise is concluded being different from that in the Principia, it will probably be acceptable to some to see one of those essays that are said by the Author (in the Principia) to be made by him towards the solution of that great and difficult problem.

Some persons will probably be ready to enquire what is the cause of this hidden virtue of gravity which is here attributed to the heavenly bodies. To this the only answer is, that this cause is as yet one of Nature's secrets: and perhaps it will ever remain so. 'Tis only plain from mathematical reasoning and undoubted observation, that such a virtue exists and operates in the

System

System of the Planets, and that according to the laws above mentioned. Till a better explication be found we may very well resolve it into the Will of the All-wise and Almighty Creator, who when he formed and disposed these bodies, by this invisible bond made them fast for ever and ever: and gave them a law which shall not be broken.

The reader is desired to observe that the references to propositions cited in the original, are omitted, so that many things he will find herein asserted, must be taken upon trust. The reason of this is, that the great work of the Principia in which those propositions were contained, appears to have been put into a very different form after this Tract was written, and many things were added and altered therein. This alteration caused so much confusion in the citations, that it was thought best to leave them all out; one or two excepted which escaped our notice, and have crept in by oversight.

OF THE SYSTEM OF THE WORLD.

That the matter of the Heavens is fluid.

IT was the ancient opinion of not a few in the earlieſt ages of philoſophy, That the fixed Stars ſtood immoveable in the higheſt parts of the world; that under the Fixed Stars the Planets were carried about the Sun; that the Earth, as one of the Planets, deſcribed an annual courſe about the Sun, while by a diurnal motion it was in the mean time revolved about its own axe; and that the Sun, as the common fire which ſerved to warm the whole, was fixed in the center of the Univerſe.

This was the philoſophy taught of old by *Philolaus*, *Ariſtarchus* of *Samos*, *Plato* in his riper years, and the whole ſect of the *Pythagoreans*. And this was the judgement of *Anaximan-*

der, more ancient than any of them, and of that wiſe king of the *Romans* *Numa Pompilius*; who as a ſymbol of the figure of the World with the Sun in the center, erected a temple in honour of *Veſta*, of a round form, and ordained perpetual fire to be kept in the middle of it.

The *Egyptians* were early obſervers of the heavens. And from them probably this philoſophy was ſpread abroad among other nations. For from them it was, and the nations about them, that the *Greeks*, a people of themſelves more addicted to the ſtudy of philology than of nature, derived their firſt as well as ſoundeſt notions of philoſophy. And in the veſtal ceremonies we may yet trace the ancient ſpirit of the *Egyptians*. For it was their way to deliver their myſteries, that is, their philoſophy of things above the vulgar way of thinking, under the veil of religious rites and hieroglyphick ſymbols.

It is not to be denied but that *Anaxagoras*, *Democritus* and others did now and then ſtart up, who would have it that the Earth poſſeſſed the center of the World, and that the Stars of all ſorts were revolved towards the weſt, about the Earth quieſcent in the center, ſome at a ſwifter, others at a ſlower rate.

However,

However, it was agreed on both ſides, that the motions of the celeſtial bodies were performed in ſpaces altogether free, and void of reſiſtance. The whim of ſolid orbs was of a later date, introduced by *Eudoxus*, *Calippus* and *Ariſtotle*; when the ancient philoſophy began to decline, and to give place to the new prevailing fictions of the *Greeks*.

But above all things the phænomena of Comets can by no means conſiſt with the notion of ſolid orbs. The *Chaldeans*, the moſt learned aſtronomers of their time, looked upon the Comets (which of ancient times before had been numbered among the celeſtial bodies) as a particular ſort of planets, which deſcribing very eccentric orbits, preſented themſelves to our view only by turns, *viz.* once in a revolution when they deſcended into the lower parts of their orbits.

And as it was the unavoidable conſequence of the hypotheſis of ſolid orbs, while it prevailed, that the Comets ſhould be thruſt down below the Moon; ſo no ſooner had the late obſervations of aſtronomers reſtored the Comets to their ancient places in the higher heavens, but theſe celeſtial ſpaces were at once cleared of the incumbrance of ſolid orbs, which

by theſe obſervations were broke into pieces and diſcarded for ever.

The principle of circular motion in free ſpaces.

WHENCE it was that the Planets came to be retained within any certain bounds in theſe free ſpaces, and to be drawn off from the rectilinear courſes, which, left to themſelves, they ſhould have purſued, into regular revolutions in curvilinear orbits, are queſtions which we do not know how the ancients explained. And probably it was to give ſome ſort of ſatisfaction to this difficulty that ſolid orbs were introduced.

The later philoſophers pretend to account for it, either by the action of certain vortices, as *Kepler* and *Des Cartes*; or by ſome other principle of impulſe or attraction, as *Borelli*, *Hook*, and others of our nation. For from the laws of motion it is moſt certain that theſe effects muſt proceed from the action of ſome force or other.

But our purpoſe is only to trace out the quantity and properties of this force from the phænomena, and to apply what we diſcover in ſome ſimple caſes, as principles, by which, in a mathematical way, we may eſtimate the effects thereof in more involved caſes. For it would be endleſs and impoſſible to bring every

par-

particular to direct and immediate obſervation.

We ſaid, *in a mathematical way*, to avoid all queſtions about the nature or quality of this force, which we would not be underſtood to determine by any hypotheſis; and therefore call it by the general name of a centripetal force, as it is a force which is directed towards ſome center; and as it regards more particularly a body in that center, we call it circum-ſolar, circum-terreſtrial, circum-jovial, and in like manner in reſpect of other central bodies.

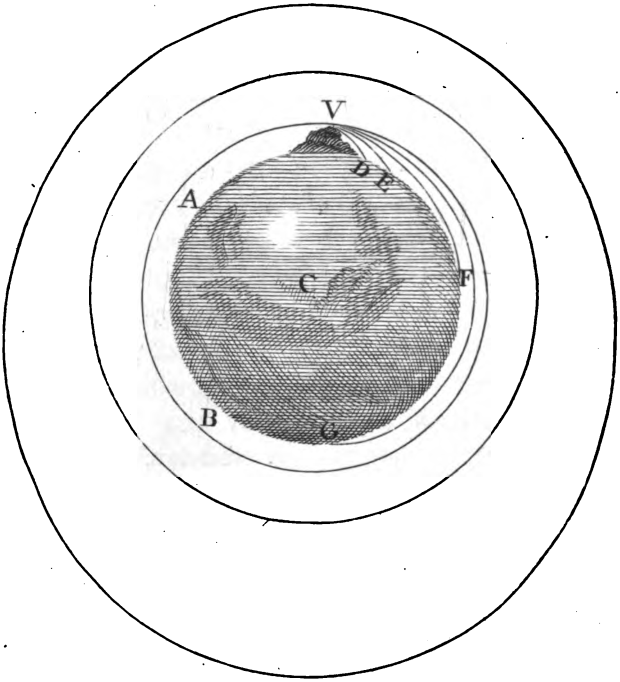
The effects of centripetal forces.

THAT by means of centripetal forces, the Planets may be retained in certain orbits, we may eaſily underſtand, if we conſider the motions of projectiles. For a ſtone projected is by the preſſure of its own weight forced out of the rectilinear path, which by the projection alone it ſhould have purſued, and made to deſcribe a curve line in the air; and through that crooked way is at laſt brought down to the ground. And the greater the velocity is with which it is projected, the farther it goes before it falls to the Earth. We may therefore ſuppoſe the velocity to be ſo encreaſed, that it would deſcribe an arc of 1, 2, 5, 10, 100,

1000 miles before it arrived at the Earth, till at laſt exceeding the limits of the Earth, it ſhould paſs quite by without touching it.

Let A F B repreſent the ſurface of the Earth, C its center, V D, V E, V F, the curve lines which a body would deſcribe if projected in an horizontal direction from the top of an high mountain, ſucceſſively with more and more velocity. And, becauſe the celeſtial motions are ſcarcely retarded by the little or no reſiſtance of the ſpaces in which they are performed ; to keep up the parity of caſes, let us ſuppoſe either that there is no air about the Earth, or at leaſt that it is endowed with little or no power of reſiſting. And for the ſame reaſon that the body projected with a leſs velocity, deſcribes the leſſer arc VD, and with a greater velocity, the greater arc V E, and augmenting the velocity, it goes farther and farther to F and G ; if the velocity was ſtill more and more augmented, it would reach at laſt quite beyond the circumference of the Earth, and return to the mountain from which it was projected.

And ſince the area's, which by this motion it deſcribes by a radius drawn to the center of the Earth, from the princi-



Page 6.

principles of geometry are demonſtrated to be proportional to the times in which they are deſcribed; its velocity, when it returns to the mountain, will be no leſs than it was at firſt; and retaining the ſame velocity it will deſcribe the ſame curve over and over, by the ſame law. *Prop.* 16.

But if we now imagine bodies to be projected in the directions of lines parallel to the horizon from greater heights, as of 5, 10, 100, 1000 or more miles, or rather as many ſemi-diameters of the Earth; thoſe bodies, according to their different velocity, and the different force of gravity in different highths, will deſcribe arcs either concentric with the Earth, or variouſly excentric, and go on revolving through the heavens in thoſe trajectories juſt as the Planets do in their orbs.

The certainty of the argument.

As when a ſtone is projected obliquely, that is, any way but in the perpendicular direction, the perpetual deflection thereof towards the Earth from the right line in which it was projected, is a proof of its gravitation to the Earth, no leſs certain than its direct deſcent when only ſuffered to fall freely from reſt; ſo the deviation of bodies, moving in free ſpaces, from rec-

tilinear paths, and perpetual deflexion therefrom towards any place, is a sure indication of the existence of some force, which from all quarters impells those bodies towards that place.

And as from the supposed existence of gravity, it necessarily follows that all bodies about the Earth must press downwards, and therefore must either descend directly to the Earth, if they are let fall from rest, or at least perpetually deviate from right lines towards the Earth, if they are projected obliquely; so from the supposed existence of a force directed to any center, it will follow by the like necessity, that all bodies, upon which this force acts, must either descend directly to that center, or at least deviate perpetually towards it from right lines, if otherwise they should have moved obliquely in these right lines.

And from the motions given we may infer the forces, or from the forces given we may determine the motions, by mathematical reasoning.

What follows from the supposed diurnal motion of the stars.

If the Earth is supposed to stand still, and the fixed Stars to be revolved in free spaces, in the space of 24 hours, it is certain the forces by which the fixed Stars are re-

tained in their orbs, are not directed to the Earth, but to the centers of the ſeveral orbs, that is, of the ſeveral parallel circles, which the fixed Stars, declining to one ſide and the other from the Equator, deſcribe daily. Alſo that by radii drawn to the centers of thoſe orbs, the fixed Stars deſcribe area's exactly proportional to the times of deſcription. Then becauſe the periodic times are equal, it follows that the centripetal forces are as the radii of the ſeveral orbs; and that they will perpetually revolve in the ſame orbs. And the like conſequences may be drawn from the ſuppoſed diurnal motion of the planets.

Prop. 1. of the ſame.

THAT forces ſhould be directed to no Body on which they phyſically depend, but to innumerable imaginary points in the axe of the Earth is an hypotheſis too incongruous. 'Tis more incongruous ſtill that thoſe forces ſhould increaſe exactly in proportion of the diſtances from this axe. For this is an indication of an increaſe to immenſity, or rather to infinity; whereas the forces of natural things commonly decreaſe in receeding from the fountain from which they flow. But what is yet more abſurd, neither are the area's, deſcribed by the ſame

The incongruous conſequences of this ſuppoſition.

ſame Star, proportional to the times, nor are its revolutions performed in the ſame orb. For as the Star recedes from the neighbouring pole, both area's and orb increaſe; and from the increaſe of the area, it is demonſtrated that the forces are not directed to the axe of the Earth. And this difficulty ariſes from the two-fold motion that is obſerved in the fixed Stars, one diurnal round the axe of the Earth, the other exceeding ſlow, round the axe of the ecliptic. And the explication thereof requires a compoſition of forces ſo perplexed and ſo variable, that it is hardly to be reconciled with any phyſical theory.

Cor. 1. *Prop.* 2.

That there is a centripetal force really directed to the center of every planet.

THAT there are centripetal forces actually directed to the bodies of the Sun, of the Earth, and other Planets, thus I infer.

The Moon revolves about our Earth, and by radii drawn to its center, deſcribes area's nearly proportional to the times in which they are deſcribed; as is evident from its velocity compared with its apparent diameter. For its motion is ſlower, when its diameter is leſs, (and therefore its diſtance greater) and its motion is ſwifter when its diameter is greater.

The

The revolutions of the ſatellits of Jupiter about that planet are more regular. For they deſcribe circles concentric with Jupiter by equable motions, as exactly as our ſenſes can diſtinguiſh.

And ſo the ſatellits of Saturn are revolved about this planet with motions nearly circular and equable, ſcarcely diſturbed by any eccentricity hitherto obſerved.

That Venus and Mercury are revolved about the Sun is demonſtrable from their Moon-like appearances. When they ſhine with a full face, they are in thoſe parts of their orbs which in reſpect of the Earth lie beyond the Sun; when they appear half full, they are in thoſe parts which lie over againſt the Sun; when horned, in thoſe parts which lie between the Earth and the Sun, and ſometimes they paſs over the Sun's diſc, when directly interpoſed between the Earth and the Sun.

And Venus, with a motion almoſt uniform, deſcribes an orb nearly circular and concentric with the Sun.

But Mercury, with a more eccentric motion, makes remarkable approaches to the Sun, and goes off again by turns; but it is always ſwifter as it is near to the Sun, and therefore by a radius drawn

to the Sun, ſtill deſcribes area's proportional to the times.

Laſtly, that the Earth deſcribes about the Sun, or the Sun about the Earth, by a radius from the one to the other, areas exactly proportional to the times, is demonſtrable from the apparent diameter of the Sun compared with its apparent motion.

Theſe are aſtronomical experiments, from which it follows by geometrical reaſoning, that there are centripetal forces, actually directed (either accurately or without conſiderable errour) to the centers of the Earth, of Jupiter of Saturn, and of the Sun. In Mercury, Venus, Mars, and the leſſer planets, where experiments are wanting, the arguments from analogy muſt be allowed in their place.

That thoſe centripetal forces decreaſe in the duplicate proportion of the diſtances from the center of every planet.

THAT thoſe forces decreaſe in the duplicate proportion of the diſtances from the center of every planet, appears by Cor. 6. Prop. 4. For the periodic times of the ſatellits of Jupiter are, one to another, in the ſeſquiplicate proportion of their diſtances from the center of this planet.

This proportion has been long ago obſerved in thoſe ſatellits. And Mr. *Flam-*

Flamsteed, who had often measured their distances from Jupiter by the micrometer, and by the eclipses of the satellits, wrote to me, that it holds to all the accuracy that possibly can be discerned by our senses. And he sent me the dimensions of their orbits taken by the micrometer and reduced to the mean distance of Jupiter from the Earth or from the Sun, together with the times of their revolutions, as follows:

	The greatest elongation of the satellits from the center of *Jupiter* as seen from the Sun.			The periodic times of their revolutions.			
	′	″	″	d.	h.	′	″
1*st.*	1	48	or 108	1	18	28	36
2*d.*	3	01	or 181	3	13	17	54
3*d.*	4	46	or 286	7	03	59	36
4*th.*	8	13½	or 493½	16	18	5	13

Whence the sesquiplicate proportion may be easily seen. For example, the $16^{d}.\ 18^{h}\ 05'.\ 13''$ is to the time $1^{d}.\ 18^{h}.\ 28'.\ 36''$ as $493\tfrac{1}{2}'' \times \sqrt{493\tfrac{1}{2}''}$ to $108'' \times \sqrt{108''}$, neglecting those small fractions which, in observing cannot be certainly determined.

Before the invention of the micrometer, the same distances were determined in semi-diameters of Jupiter, thus:

Dis-

Distance of the	1st.	2d.	3d.	4th.
By *Galilæo*	6	10	16	28
Simon Marius	6	10	16	26
Cassini	5	8	13	23
Borelli more exactly	$5\frac{2}{3}$	$8\frac{2}{3}$	14	$24\frac{2}{3}$

After the invention of the micrometer.

By *Townley*	5,51	8,78	13,47	24,72
Flamsteed	5,31	8,55	13,98	24,23
More accurately by the eclipses	5,578	8,876	14,159	24,903

And the periodic times of those satellits, by the observations of Mr. *Flamsteed*, are $1^{d}.\ 18^{h}.\ 28'\ 36''$ | $3^{d}.\ 13^{h}.\ 17'\ 54''$ | $7^{d}.\ 3^{h}.\ 59'\ 36''$ | $16^{d}.\ 18^{h}.\ 5'\ 13''$ | as above.

And the distances thence computed are 5,578 | 8,878 | 14,168 | 24,968 | accurately agreeing with the distances by observation.

Cassini assures us that the same proportion is observed in the circum-saturnal planets. But a longer course of observations is required before we can have a certain and accurate theory of those planets.

In

In the circum-ſolar planets, Mercury and Venus, the ſame proportion holds with great accuracy according to the dimenſions of their orbs, as determined by the obſervations of the beſt Aſtronomers.

That the ſuperiour planets are revolved about the Sun, and by radii drawn to the Sun, deſcribe area's proportional to the times.

That Mars is revolved about the Sun, is demonſtrated from the phaſes which it ſhews, and the proportion of its apparent diameters. For from its appearing full near conjunction with the Sun and gibbous in its quadratures, it is certain that it ſurrounds the Sun.

And ſince its diameter appears about five times greater, when in oppoſition to the Sun, than when in conjunction therewith, and its diſtance from the Earth is reciprocally as its apparent diameter, that diſtance will be about 5 times leſs, when in oppoſition to, than when in conjunction with, the Sun. But in both caſes its diſtance from the Sun will be near about the ſame with the diſtance which is inferred from its gibbous appearance in the quadratures. And as it encompaſſes the Sun at almoſt equal diſtances, but in reſpect of the Earth is very unequally diſtant; ſo by radii drawn to the Sun it deſcribes

area's

area's nearly uniform, but by radii drawn to the Earth, it is sometimes swift, sometimes stationary, and sometimes retrograde.

That Jupiter, in a higher orb than Mars, is likewise revolved about the Sun, with a motion nearly equable, as well in distance as in the area's described, I infer thus.

Mr. *Flamsteed* assured me by letters, that all the eclipses of the innermost satellite, which hitherto have been well observed, do agree with his theory so nearly, as never to differ therefrom by two minutes of time; that in the outmost, the errour is little greater; in the outmost but one, scarcely three times greater; that in the innermost but one, the difference is indeed much greater, yet so as to agree as nearly with his computations, as the Moon does with the common tables. And that he computes those eclipses only from the mean motions corrected by the equation of light discovered and introduced by Mr. *Romer*. Supposing then that the theory differs by a less errour than that of 2′ from the motion of the outmost satellite as hitherto described; and taking as the periodic time $16^d. 18^h. 5' 13''$ to 2′ in time, so is the whole circle or 360° to the

the arc 1′ 48″; the errour of Mr. *Flamsteed*'s computation reduced to the satellite's orbit, will be less than 1′ 48″, that is, the longitude of the satellite, as seen from the center of Jupiter, will be determined with a less errour than 1′ 48″. But when the satellite is in the middle of the shadow, that longitude is the same with the heliocentric longitude of Jupiter. And therefore the hypothesis which Mr. *Flamsteed* follows, *viz.* the *Copernican* as improved by *Kepler*, and (as to the motion of Jupiter) lately corrected by himself, rightly represents that longitude within a less errour than 1′ 48″. But by this longitude, together with the geocentric longitude, which is always easily found, the distance of Jupiter from the Sun is determined: Which must therefore be the very same with that which the hypothesis exhibits. For that greatest errour of 1′ 48″ that can happen in the heliocentric longitude is almost insensible, and quite to be neglected, and perhaps may arise from some yet undiscovered eccentricity of the satellite. But when both longitude and distance are rightly determined, it follows of necessity, that Jupiter by radii drawn to the Sun, describes area's so

C con-

conditioned as the hypothesis requires, that is, proportional to the times.

And the same thing may be concluded of Saturn from his satellite by the observations of Mr. *Huygens* and Dr. *Halley*; though a longer series of observations is yet wanting to confirm the thing, and to bring it under a sufficiently exact computation.

That the force which governs the superiour Planets is directed, not to the Earth but to the Sun.

For if Jupiter was viewed from the Sun, it would never appear retrograde nor stationary, as it is seen sometimes from the Earth, but always to go forward with a motion nearly uniform. And from the very great inequality of its apparent geocentric motion, we infer that the force by which Jupiter is turned out of a rectilinear course and made to revolve in an orb, is not directed to the center of the Earth. And the same argument holds good in Mars and in Saturn. Another center is therefore to be looked for, about which the area's described by radii intervening, may be equable. And that this is the Sun, we have proved already in Mars and Saturn nearly, but accurately enough in Jupiter. It may be alledged that the Sun and Planets are impelled by

by ſome other force, equally and in the direction of parallel lines. But by ſuch a force no change would happen in the ſituation of the Planets one to another, nor any ſenſible effect follow; but our buſineſs is with the cauſes of ſenſible effects. Let us therefore neglect every ſuch force as imaginary and precarious, and of no uſe in the phænomena of the heavens; and the whole remaining force by which Jupiter is impelled, will be directed to the center of the Sun. The argument from analogy proves the ſame thing in Mars and Saturn; and in Saturn, it may be made out directly from its ſatellite by the obſervations of Mr. *Huygens* and Dr. *Halley*, though a longer ſeries of obſervations is required for the confimation of it.

That the circum-ſolar force throughout all the regions of the Planets decreaſeth in the duplicate proportion of the diſtances from the Sun.

THE diſtances of the Planets from the Sun come out the ſame, whether, with *Tycho*, we place the Earth in the center of the ſyſtem, or the Sun with *Copernicus*: and we have already proved that theſe diſtances are true in Jupiter.

Kepler and *Bullialdus* have, with great care, determined the diſtances of the planets from the Sun. And hence it

it is that their tables agree beſt with the heavens. And in all the Planets, in Jupiter and Mars, in Saturn and the Earth, as well as in Venus and Mercury, the cubes of their diſtances are as the ſquares of their periodic times; and therefore the centripetal circum-ſolar force, throughout all the planetary regions, decreaſes in the duplicate proportion of the diſtances from the Sun. In examining this proportion, we are to uſe the mean diſtances, or the tranſverſe ſemi-axes of the orbits, and to neglect thoſe little fractions, which, in defining the orbits, may have aroſe from the inſenſible errours of obſervation, or may be aſcribed to other cauſes which we ſhall afterwards explain. And thus we ſhall always find the ſaid proportion to hold exactly. For the diſtances of Saturn, Jupiter, Mars, the Earth, Venus and Mercury from the Sun drawn from the obſervations of Aſtronomers, are, according to the computation of *Kepler*, as the numbers 951000,519650,152350, 100000,72400,38806; by the computation of *Bullialdus*, as the numbers, 954198,522520,152350, 100000, 72398, 38585; and from the periodic times they come out 953806,520116,152399, 100000, 72333, 38710. Their diſtances,

ces, according to *Kepler* and *Bullialdus*, ſcarcely differ by any ſenſible quantity, and where they differ moſt, the diſtances drawn from the periodic times fall in between them.

That the circum-terreſtrial force decreaſes in the duplicate proportion of the diſtances from the Earth proved in the hypotheſis of the Earth's being at reſt.

THAT the circum-terreſtrial force likewiſe decreaſes in the duplicate proportion of the diſtances, thus I infer.

The mean diſtance of the Moon from the center of the Earth, is in ſemi-diameters of the Earth, according to *Ptolomy*, *Kepler* in his *Ephemerides*, *Bullialdus*, *Hevelius* and *Ricciolus*, 59; according to *Flamſteed* 59$\frac{1}{3}$, according to *Tycho* 56$\frac{1}{2}$, to *Vendelin* 60, to *Copernicus* 60$\frac{1}{3}$, to *Kircher* 62$\frac{1}{2}$.

But *Tycho* and all that follow his tables of refraction, making the refractions of the Sun and Moon (altogether againſt the nature of light) to exceed thoſe of the fixed Stars, and that by about four or five minutes in the horizon, did thereby augment the horizontal parallax of the Moon, by about the like number of minutes, that is, by about the 12th or 15th part of the whole parallax. Correct this errour, and the diſtance will become 60 or 61 ſemi-diame-

ters of the Earth, nearly agreeing with what others have determined.

Let us then aſſume the mean diſtance of the Moon, 60 ſemi-diameters of the Earth, and its periodic time in reſpect of the fixed Stars, 27^{d}. 7^{h}. 43′, as Aſtronomers have determined it. And a body revolved in our air, near the ſurface of the Earth ſuppoſed at reſt, by means of a centripetal force, which ſhould be, to the ſame force at the diſtance of the Moon, in the reciprocal duplicate proportion of the diſtances from the center of the Earth, that is, as 3600 to 1; would (ſecluding the reſiſtance of the air) compleat a revolution in 1 h. 24′, 27″.

Suppoſe the circumference of the Earth to be 123249600 *Paris* feet, as has been determined by the late menſuration of the *French*: and the ſame body, deprived of its circular motion, and falling by the impulſe of the ſame centripetal force as before, would, in one ſecond of time, deſcribe 15 $\frac{1}{12}$ *Paris* feet.

This we infer by a calculus, and it agrees with what we obſerve in all bodies about the Earth. For by the experiments of pendulums, and a computation

tion raised thereon, Mr. *Huygens* has demonstrated that bodies falling by all that centripetal force, with which (of whatever nature it is) they are impelled near the surface of the Earth, do, in one second of time, describe 15 $\frac{1}{12}$ *Paris* feet.

The same proved in the hypothesis of the Earth's motion.

But if the Earth is supposed to move, the Earth and Moon together will be revolved about their common center of gravity. And the Moon will in the same periodic time $27^{d}. 7^{h}. 43'$, with the same circum-terrestrial force, diminished in the duplicate proportion of the distance, describe an orbit, whose semi-diameter is to the semi-diameter of the former orbit, that is, to 60 semi-diameters of the Earth, the sum of both the bodies of the Earth and Moon to the first of two mean proportionals between this sum and the body of the Earth; that is, if we suppose the Moon (on account of its mean apparent diameter 31 $\frac{1}{2}'$) to be about $\frac{1}{42}$ of the Earth, as 43 to $\sqrt{c.\ \overline{42 \times 43|^{2}}}$, or as about 128 to 127. And therefore the semi-diameter of the orbit, that is, the distance between the centers of the Moon and Earth, will in this case be 60 $\frac{1}{2}$ semidiameters of the Earth, almost the same with that assigned by *Copernicus*,

 which

which the *Tychonic* obſervations by no means diſprove. And therefore the duplicate proportion of the decrement of the force holds good in this diſtance. I have neglected the increment of the orbit, which ariſes from the action of the Sun, as inconſiderable. But if that is ſubducted, the true diſtance will remain, about $60\frac{4}{9}$ ſemidiameters of the Earth.

The decrement of the forces in the duplicate proportion of the diſtances from the Earth and Planets, proved from the eccentricity of the Planets, and the very ſlow motion of their apſes.

But further, this proportion of the decrement of the forces is confirmed from the eccentricity of the Planets, and the very ſlow motion of their apſes. For in no other proportion, could the circum-ſolar planets, once in every revolution deſcend to their leaſt and once aſcend to their greateſt diſtance from the Sun, and the places of thoſe diſtances remain immovable. A ſmall errour from the duplicate proportion, would produce a motion of the apſes, conſiderable in every revolution, but in many enormous.

But now after innumerable revolutions, hardly any ſuch motion has been perceived in the orbs of the circum-ſolar planets. Some Aſtronomers affirm, that there is no ſuch motion, others reckon it no greater than what may eaſily

ariſe

arise from the causes hereafter to be assigned, and is of no moment in the present question.

We may even neglect the motion of the Moon's apse, which is far greater than in the circum-solar planets, amounting in every revolution to three degrees. And from this motion it is demonstrable, that the circum-terrestrial force decreases in no less than the duplicate, but far less than the triplicate proportion of the distance. For if the duplicate proportion was gradually changed into the triplicate, the motion of the apse would thereby increase to infinity; and therefore, by a very small mutation, would exceed the motion of the Moon's apse. This slow motion arises from the action of the circum-solar force, as we shall afterwards explain. But secluding this cause, the apse or apogeon of the Moon will be fixed, and the duplicate proportion of the decrease of the circum-terrestrial force in different distances from the Earth, will accurately take place.

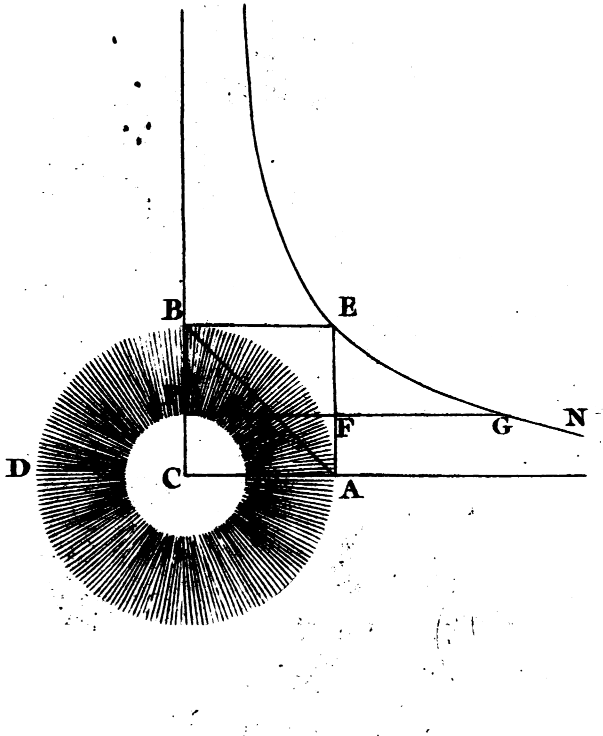
Now that this proportion has been established, we may compare the forces of the several planets among themselves.

The quantity of the forces tending towards the several planets. The circum-solar very great.

In the mean distance of Jupiter

piter from the Earth, the greateſt elongation of the outmoſt ſatellite from Jupiter's center (by the obſervations of Mr. *Flamſteed*) is 8′ 13″. And therefore the diſtance of the ſatellite from the center of Jupiter, is to the mean diſtance of Jupiter from the center of the Sun, as 124 to 52012, but to the mean diſtance of Venus from the center of the Sun, as 124 to 7234. And their periodic times are $16\frac{3}{4}{}^{d}$. and $224\frac{2}{3}{}^{d}$. And from hence (according to *Cor.* 2. *Prop.* 4.) dividing the diſtances by the ſquares of the times, we infer that the force by which the ſatellite is impelled towards Jupiter, is to the force by which Venus is impelled towards the Sun, as 442 to 143. And if we diminiſh the force, by which the ſatellite is impelled in the duplicate proportion of the diſtance 124 to 7234, we ſhall have the circum-jovial force, in the diſtance of Venus from the Sun, to the circum-ſolar force by which Venus is impelled, as $\frac{13}{100}$ to 143, or as 1 to 1100. Wherefore at equal diſtances, the circum-ſolar force is 1100 times greater than the circum-jovial.

And by the like computation, from the periodic time of the ſatellite of Saturn $15^{d}. 22\frac{2}{3}{}^{h}$. and its greateſt elongation from



Page 29.

fromSaturn,while that planet is in its mean diſtance from us, $3' \ 20''$; it follows that the diſtance of this ſatellite from Saturn's center, is to the diſtance of Venus from the Sun, as $92\frac{2}{5}$ to 7234; and from thence that the abſolute circum-ſolar force is 2360 times greater than the abſolute circum-ſaturnial.

The circum-terreſtrial force very ſmall.

FROM the regularity of the heliocentric and irregularity of the geocentric motions of Venus, of Jupiter, and the other planets, it is evident that the circum-terreſtrial force, compared with the circum-ſolar, is very ſmall.

Ricciolus and *Vendelin* have ſeverally tried to determine the Sun's parallax, from the Moon's dichotomies obſerved by the teleſcope, and they agree that it does not exceed half a minute.

Kepler from *Tycho's* obſervations and his own, found the parallax of Mars inſenſible, even in oppoſition to the Sun, when that parallax is ſomething greater than the Sun's.

Flamſteed attempted the ſame parallax with the micrometer in the perigeon poſition of Mars, but never found it above $25''$; and thence concluded the Sun's parallax at moſt $10''$.

Whence

Whence it follows that the diſtance of the Moon from the Earth bears no greater proportion to the diſtance of the Earth from the Sun, than 29 to 10000; nor to the diſtance of Venus from the Sun, than 29 to 7233.

From which diſtances, together with the periodic times, by the method above explained it is eaſy to infer, That the abſolute circum-ſolar force is greater than the abſolute circum-terreſtrial force at leaſt 229400 times.

And though we were only certain, from the obſervations of *Riccioius* and *Vendelin*, that the Sun's parallax was leſs than half a minute, yet from this it will follow, that the abſolute circum-ſolar force exceeds the abſolute circum-terreſtrial force 8500 times.

The apparent diameters of the planets.

By the like Computations I happened to diſcover an analogy that is obſerved between the forces and the bodies of the Planets. But before I explain this analogy, the apparent diameters of the Planets in their mean diſtances from the Earth, muſt be firſt determined.

Mr. *Flamſteed*, by the micrometer, meaſured the diameter of Jupiter 40″ or 41″, the diameter of Saturn's ring 50″, and

and the diameter of the Sun about 32′. 13″.

But the diameter of Saturn is to the diameter of the ring, according to Mr. *Huygens* and Dr. *Halley*, as 4 to 9; according to *Galletius*, as 4 to 10; and according to *Hook* (by a teleſcope of 60 feet) as 5 to 12. And from the mean proportion 5 to 12, the diameter of Saturn's body is inferred about 21″.

The correction of the apparent diameters.

SUCH as we have ſaid are the apparent magnitudes. But becauſe of the unequal refrangibility of light, all lucid points are dilated by the teleſcope, and in the focus of the objective glaſs poſſeſs a circular ſpace, whoſe breadth is about the 50th part of the aperture of the glaſs.

It is true, that towards the circumference, the light is ſo rare as hardly to move the ſenſe, but towards the middle, where it is of greater denſity, and is ſenſible enough, it makes a ſmall lucid circle, whoſe breadth varies according to the ſplendour of the lucid point, but is generally about the 3d, or 4th, or 5th part of the breadth of the whole.

Let A B D repreſent the circle of the whole light, P Q the ſmall circle of the denſer and clearer light, C the center of both;

both; C A, C B ſemi-diameters of the greater circle containing a right angle at C; A C B E the ſquare comprehended under theſe ſemi-diameters, A B the diagonal of that ſquare; E G H an hyperbola with the center C and aſymptotes C A, C B; P G a perpendicular erected from any point P of the line B C, and meeting the hyperbola in G, and the right lines A B, A E in K and F: and the denſity of the light in any place P, will, by my computation, be as the line F G, and therefore at the center infinite, but near the circumference very ſmall. And the whole light within the ſmall circle P Q is to the whole without, as the area of the quadrilateral figure C A K P to the triangle P K B. And we are to underſtand the ſmall circle P Q to be there terminated, where F G the denſity of the light begins to be leſs than what is required to move the ſenſe.

Hence it was that at the diſtance of 191382 feet, a fire of 3 feet in diameter, through a teleſcope of 3 feet, appeared to Mr. *Picart* of 8″. in breadth, when it ſhould have appeared only of 3″. 14‴. And hence it is that the brighter fixed Stars appear through the teleſcope, as of 5″ or 6″ in diameter, and that with a good

good full light, but with a fainter light they appear to run out to a greater breadth. Hence likewiſe it was that *Hevelius*, by diminiſhing the aperture of the teleſcope, did cut off a great part of the light towards the circumference, and brought the diſc of the Star to be more diſtinctly defined, which though hereby diminiſhed, did yet appear as of 5″ or 6″ in diameter. But Mr. *Huygens* only by clouding the eye-glaſs with a little ſmoak, did ſo effectually extinguiſh this ſcattered light, that the fixed Stars appeared as meer points, void of all ſenſible breadth. Hence alſo it was that Mr. *Huygens*, from the breadth of bodies interpoſed to intercept the whole light of the Planets, reckoned their diameters greater than others have meaſured them by the micrometer. For the ſcattered light which could not be ſeen before for the ſtronger light of the Planet, when the Planet is hid appears every way further ſpread. Laſtly from hence it is that the Planets appear ſo ſmall in the diſc of the Sun, being leſſened by the dilated light. For to *Hevelius*, *Galletius*, and Dr. *Halley*, Mercury did not ſeem to exceed 12″ or 15″. And Venus appeared to Mr. *Crabtrie* only 1′. 3″. to *Horrox* but 1′. 12″, though by

by the menſurations of *Hevelius* and *Hugenius* without the Sun's diſc, it ought to have been ſeen at leaſt 1'. 24". Thus the apparent diameter of the Moon, which in 1684, a few days both before and after the Sun's eclipſe, was meaſured at the obſervatory of *Paris* 31'. 30"; in the eclipſe it ſelf did not ſeem to exceed 30' or 30'. 05". And therefore the diameters of the Planets are to be diminiſhed, when without the Sun, and to be augmented when within it by ſome ſeconds. But the errours ſeem to be leſs than uſual in the menſurations that are made by the micrometer. So from the diameter of the ſhadow, determined by the eclipſes of the ſatellites, Mr. *Flamſteed* found that the diameter of Jupiter, was to the greateſt elongation of the outmoſt ſatellite as 1 to 24,903. Wherefore ſince that elongation is 8'. 13", the diameter of Jupiter will be $39\frac{1}{2}''$; and rejecting the ſcattered light, the diameter, found by the micrometer 40" or 41", will be reduced to $39\frac{1}{2}$. And the diameter of Saturn 21", is to be diminiſhed by the like correction, and to be reckoned 20" or ſomething leſs. But (if I am not miſtaken) the diameter of the Sun, be-

cauſe

cause of its stronger light is to be diminish'd something more, and to be reckon'd about 32′. or 32′. 6″.

Why the density is greater in some of the Planets, and less in others; but the forces in all are as their quantities of matter.

THAT so near an analogy should be found among bodies of magnitudes so different, is not without some mystery.

It may be that the remoter Planets, for want of heat, have not those metallic substances and ponderous minerals with which our Earth abounds; and that the bodies of Venus and Mercury, as they are more exposed to the Sun's heat, are also harder baked and more compact.

For from the experiment of the burning glass, we see that the heat increases with the density of light. And this density increases in the reciprocal duplicate proportion of the distance from the Sun. From whence the Sun's heat in Mercury is proved to be seven-fold its heat in our summer seasons. But with this heat our water boyls; and those heavy fluids, quicksilver and the spirits of vitriol, gently evaporate, as I have tryed by the thermometer. And therefore there can be no fluids in Mercury, but what are heavy, and able to bear a great heat, and from which sub-

ſtances of great denſity may be nouriſh'd.

And why not? if God has plac'd different bodies at different diſtances from the Sun, ſo as the denſer bodies always poſſeſs the nearer places, and each body enjoys a degree of heat ſuitable to its condition, and proper for its nouriſhment. From this conſideration it will beſt appear that the weights of all the Planets are one to another as their forces.

But I ſhould be glad the diameters of the Planets were more accurately meaſured. And that may be done, if a lamp, ſet at a great diſtance, is made to ſhine through a circular hole, and both the hole and the light of the lamp are ſo diminiſhed that the ſpectrum may appear through the teleſcope juſt like the Planet, and may be defined by the ſame meaſure. Then the diameter of the hole will be to its diſtance from the objective glaſs, as the true diameter of the Planet to its diſtance from us. The light of the lamp may be diminiſhed by the interpoſition either of pieces of cloath, or of ſmoaked glaſs.

Another analogy between the forces and bodies; proved in the celestial bodies.

Of kin to the analogy we have been describing, there is another observed betwen the forces and the bodies attracted. Since the action of the centripetal force upon the Planets decreases in the duplicate proportion of the distance, and the periodic time increases in the sesquiplicate thereof, it is evident that the actions of the centripetal force, and therefore the periodic times, would be equal in equal Planets, at equal distances from the Sun; and in equal distances of unequal Planets, the total actions of the centripetal force would be as the bodies of the Planets. For if the actions were not proportional to the bodies to be moved, they could not equally retract these bodies from the tangents of their orbs, and force them to revolve in equal orbs, in equal times. Nor could the motions of the satellites of Jupiter be so regular, if it was not that the circum-solar force was equally exerted upon Jupiter and all its satellites in proportion of their several weights. And the same thing is to be said of Saturn in respect of its satellite, and of our Earth in respect of the Moon. And therefore at equal distances, the action of the centripetal force is equal upon all the Planets, in

proportion of their bodies, or of the quantities of matter in their ſeveral bodies: and for the ſame reaſon muſt be the ſame upon all the particles of the ſame ſize, of which the planet is compoſed. For if the action was greater upon ſome ſort of particles than upon others then in proportion to their quantity of matter, it would be alſo greater or leſs, upon the whole Planets, not in proportion of the quantity only, but likewiſe of the ſort of the matter more copiouſly found in one, and more ſparingly in another.

Proved in terreſtrial bodies.

In ſuch bodies as are found on our Earth of very different ſorts, I examined this analogy with great accuracy.

If the action of the circumterreſtrial force is proportional to the bodies to be moved, it will move them with equal velocity in equal times, and will make all bodies let fall to deſcend through equal ſpaces in equal times; and all bodies hung by equal threads, to vibrate in equal times. If the action of the force was greater, the times would be leſs. If that was leſs, theſe would be greater.

But it has been long ago obſerved by others, that (allowance being made for the

the ſmall reſiſtance of the Air) all bodies deſcend through equal ſpaces in equal times. And, by the help of pendulums, that equality of times may be diſtinguiſh'd to great exactneſs.

I tryed the thing in gold, ſilver, lead, glaſs, ſand, common ſalt, wood, water and wheat. I provided two equal wooden boxes. I filled the one with wood, and ſuſpended an equal weight of gold (as exactly as I could) in the center of oſcillation of the other. The boxes hung by equal threads of 11 feet, made a couple of pendulums perfectly equal in weight and figure, and equally expoſed to the reſiſtance of the air. And placing the one by the other, I obſerved them to play together, forwards and backwards, for a long while, with equal vibrations. And therefore the quantity of matter in the gold was to the quantity of matter in the wood, as the action of the motive force upon all the gold, to the action of the ſame upon all the wood, that is, as the weight of the one to the weight of the other.

And by theſe experiments, in bodies of the ſame weight, I could have diſcovered a difference of matter, leſs than the thouſandth part of the whole.

The affinity of those analogies.

SINCE the action of the centripetal force upon the bodies attracted, is, at equal distances, proportional to the quantities of matter in those bodies; reason requires that it should be also proportional to the quantity of matter in the body attracting.

For all action is mutual, and makes the bodies mutually to approach one to the other, and therefore must be the same in both bodies. It is true that we may consider one body as attracting, another as attracted. But this distinction is more mathematical than natural. The attraction is really common of either to other, and therefore of the same kind in both.

And coincidence.

AND hence it is that the attractive force is found in both. The Sun attracts Jupiter and the other Planets. Jupiter attracts its satellites. And for the same reason, the satellites act as well one upon another as upon Jupiter, and all the Planets mutually one upon another.

And though the mutual actions of two Planets may be distinguished and considered as two, by which each attracts the other; yet as those actions are

are intermediate, they don't make two, but one operation between two terms. Two bodies may be mutually attracted, each to the other, by the contraction of a cord interposed. There is a double cause of action, to wit, the disposition of both bodies, as well as a double action in so far as the action is considered as upon two bodies. But as betwixt two bodies it is but one single one. 'Tis not one action by which the Sun attracts Jupiter, and another by which Jupiter attracts the Sun. But it is one action by which the Sun and Jupiter mutually endeavour to approach each the other. By the action with which the Sun attracts Jupiter, Jupiter and the Sun endeavour to come nearer together, and by the action, with which Jupiter attracts the Sun, likewise Jupiter and the Sun endeavour to come nearer together. But the Sun is not attracted towards Jupiter by a two-fold action, nor Jupiter by a two-fold action towards the Sun: but 'tis one single intermediate action, by which both approach nearer together.

Thus iron draws the load-stone, as well as the load-stone draws the iron. For all iron in the neighbourhood of the load-stone draws other iron. But the

action betwixt the load-ſtone and iron is ſingle, and is conſidered as ſingle by the philoſophers. The action of iron upon the load-ſtone is indeed the action of the load ſtone betwixt it ſelf and the iron, by which both endeavour to come nearer together. And ſo it manifeſtly appears For if you remove the load-ſtone, the whole force of the iron almoſt ceaſes.

In this ſenſe it is that we are to conceive one ſingle action to be exerted betwixt two Planets, ariſing from the conſpiring natures of both. And this action ſtanding in the ſame relation to both, if it is proportional to the quantity of matter in the one, it will be alſo proportional to the quantity of matter in the other.

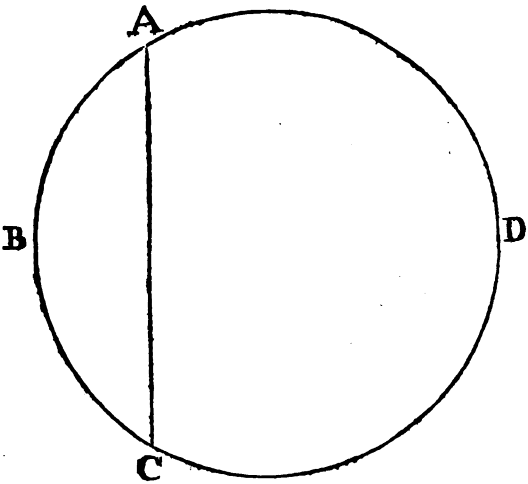
That the forces of ſmall bodies are inſenſible.

PERHAPS it may be objected, that according to this philoſophy, all bodies ſhould mutually attract one another, contrary to the evidence of experiments in terreſtrial bodies. But I anſwer, that the experiments in terreſtrial bodies come to no account. For the attractions of homogeneous ſpheres near their ſurfaces, are as their diameters. Whence a ſphere of one foot in diameter, and of a like

nature

nature to the Earth, would attract a small body plac'd near its surface with a force about 20000000 times less, than the Earth would do if placed near its surface. But so small a force could produce no sensible effect. If two such spheres were distant but by ¼ of an inch, they would not even in spaces void of resistance, come together by the force of their mutual attraction in less than a months time. And less spheres will come together at a rate yet slower, viz. in the proportion of their diameters. Nay whole mountains will not be sufficient to produce any sensible effect. A mountain of an hemispherical figure, three miles high, and six broad, will not by its attraction, draw the pendulum 2 minutes out of the true perpendicular. And 'tis only in the great bodies of the Planets that these forces are to be perceiv'd, unless we may reason about smaller bodies in manner following.

LET



Which notwithstanding, there are forces tending towards all terrestrial bodies, proportional to their quantities of matter.

LET ABCD represent the globe of the Earth, cut, by any plane AC, into two parts ACB, and ACD. The part ACB bearing upon the part ACD presses it with its whole weight. Nor can the part ACD sustain this pressure and continue unmoved, if it is not opposed by an equal contrary pressure. And therefore the parts equally press each other by their weights, that is, equally attract each other, and if separated and let go would fall towards each other with velocities reciprocally as the bodies. All which we may try and see in the load-stone, whose

whose attracted part does not propell the part attracting, but is only stopped and sustained thereby.

Suppose now that A C B represents some small body on the Earth's surface; and because the mutual attractions of this particle and of the remaining part of the Earth towards each other, are equal; but the attraction of the particle towards the Earth (or its weight) is as the matter of the particle (as we have proved by the experiment of the pendulums) the attraction of the Earth towards the particle, will likewise be as the matter of the particle. And therefore the attractive forces of all terrestrial bodies, will be as their several quantities of matter.

Prov'd that the same forces tend towards the celestial bodies.

The forces, which are as the matter in terrestrial bodies of all forms, and therefore are not mutable with the forms, must be found in all sorts of bodies whatsoever, celestial as well as terrestrial, and be in all proportional to their quantities of matter, because among all there is no difference of substance, but of modes and forms only. But in the celestial bodies, the same thing is likewise proved thus. We have shewed, that the action

of the circumſolar force upon all the Planets (reduced to equal diſtances) is as the matter of the Planets; That the action of the circumjovial force upon the ſatellites of Jupiter obſerves the ſame law; and the ſame thing is to be ſaid of the attraction of all the Planets towards every Planet. But thence it follows that their attractive forces are as their ſeveral quantities of matter.

That from the ſurfaces of the Planets, reckoning outward, their forces decreaſe in the duplicate; but, reckoning inward, in the ſimple proportion of the diſtances from their centers.

As the parts of the Earth mutually attract one another, ſo do thoſe of all the Planets. If Jupiter and its ſatellites were brought together, and formed into one globe; without doubt, they would continue mutually to attract one another as before. And on the other hand, if the body of Jupiter was broke into more globes, to be ſure, theſe would no leſs attract one another than they do the ſatellites now. From theſe attractions it is that the bodies of the Earth, and all the Planets affect a ſpherical figure, and that their parts cohere, and are not diſperſed through the Ether. But we have before proved that theſe forces ariſe from the univerſal nature of matter, and that therefore the force of any whole globe is made up of the ſeveral forces

forces of all its parts. And from thence it follows, that the force of every particle decreaſes in the duplicate proportion of the diſtance from that particle; and that the force of an entire globe, reckoning from the ſurface outwards, decreaſes in the duplicate, but reckoning inwards, in the ſimple proportion of the diſtances, from the centres, if the matter of the globe be uniform. And though the matter of the globe, reckoning from the center towards the ſurface, is not uniform; yet the decreaſe in the duplicate proportion of the diſtance outwards would take place, provided that difformity is ſimilar in places round about at equal diſtances from the center. And two ſuch globes will attract one the other with a force decreaſing in the duplicate proportion of the diſtance between their centers.

The quantities of the forces and of the motions ariſing in the ſeveral caſes.

WHEREFORE the abſolute force of every globe is as the quantity of matter which the globe contains. But the motive force by which every globe is attracted towards another, and which, in terreſtrial bodies, we commonly call their weight, is as the content under the quanties of matter in both globes apply-

ed

ed to the ſquare of the diſtance between their centers; to which force the quantity of motion, by which each globe in a given time will be carried towards the other, is proportional. And the accelerative force, by which every globe according to its quantity of matter is attracted towards another, is as the quantity of matter in that other globe applyed to the ſquare of the diſtance between the centers of the two; to which force, the velocity, by which the attracted globe will, in a given time, be carried towards the other, is proportional. And from theſe principles well underſtood, it will be now eaſy to determine the motions of the celeſtial bodies among themſelves.

That all the Planets revolve about the Sun.

FROM comparing the forces of the Planets one with another, we have above ſeen that the circum-ſolar does more than a thouſand times exceed all the reſt. But by the action of a force ſo great, it is unavoidable but that all bodies within, nay and far beyond, the bounds of the planetary ſyſtem, muſt deſcend directly to the Sun, unleſs by other motions they are impelled towards other parts. Nor is our Earth to be excluded

from

from the number of ſuch bodies. For certainly the Moon is a body of the ſame nature with the Planets, and ſubject to the ſame attractions with the other Planets, ſeeing it is by the circumterreſtrial force that it is retained in its orbit. But that the Earth and Moon are equally attracted towards the Sun, we have above proved: We have likewiſe before proved, that all bodies are ſubject to the ſame common laws of attraction. Nay, ſuppoſing any of thoſe bodies to be deprived of its circular motion about the Sun, by having its diſtance from the Sun we may find in what ſpace of time it would, in its deſcent, arrive at the Sun; to wit, in half that periodic time, in which the body might be revolved at one half of its former diſtance; or, in a ſpace of time that is to the periodic time of the planet as 1 to $4\sqrt{2}$. As that Venus in its deſcent would arrive at the Sun in the ſpace of 40 days, Jupiter in the ſpace of two years and one month; and the Earth and Moon together in the ſpace of 66 days and 19 hours. But ſince no ſuch thing happens, it muſt needs be that thoſe bodies are moved towards other parts. Nor is every motion ſufficient for this purpoſe. To hinder ſuch

a deſcent, a due proportion of velocity is required. And hence depends the force of the argument drawn from the retardation of the motions of the Planets. Unleſs the circum-ſolar force decreaſed in the duplicate ratio of their increaſing ſlowneſs, the exceſs thereof would force thoſe bodies to deſcend to the Sun. For inſtance, if the motion (*cæteris paribus*) was retarded by one half, the planet would be retained in its orb by one fourth of the former circum-ſolar force, and by the exceſs of the other three fourths would deſcend to the Sun. And therefore the Planets (Saturn, Jupiter, Mars, Venus, and Mercury) are not really retarded in their perigees, nor become really ſtationary or regreſſive with ſlow motions. All theſe are but apparent, and the abſolute motions, by which the Planets continue to revolve in their orbits, are always direct and nearly equable. But that ſuch motions are performed about the Sun, we have already proved; and therefore the Sun, as the center of the abſolute motions, is quieſcent. For we can by no means allow quieſcence to the Earth, leaſt the Planets in their perigees ſhould indeed be truly retarded, and become truly ſtationary and regreſ-

ſive,

ſive, and ſo for want of motion ſhould deſcend to the Sun. But further, ſince the Planets (Venus, Mars, Jupiter and the reſt) by radii drawn to the Sun deſcribe regular orbits, and area's (as we have ſhewed) nearly and to ſenſe proportional to the times; it follows that the Sun is moved with no notable force, unleſs perhaps with ſuch as all the Planets are equally moved with, according to their ſeveral quantities of matter, in parallel lines, and ſo the whole ſyſtem is transferred in right lines. Reject that tranſlation of the whole ſyſtem, and the Sun will be almoſt quieſcent in the center thereof. If the Sun was revolved about the Earth, and carried the other Planets round about it ſelf, the Earth ought to attract the Sun with a great force, but the circum-ſolar Planets with no force producing any ſenſible effect, which is abſurd. Add to this, that if hitherto the Earth, becauſe of the gravitation of its parts, has been placed by moſt authors in the lowermoſt region of the Univerſe; now for better reaſon, the Sun poſſeſſed of a centripetal force exceeding our terreſtrial gravitation a thouſand times and more, ought to be depreſſed into the lowermoſt place, and to be held for the center of the ſyſ-

tem. And thus the true disposition of the whole system will be more fully and more exactly understood.

That the common center of gravity of all the Planets is quiescent. That the Sun is agitated with a very slow motion. This motion defined.

BECAUSE the fixed Stars are quiescent one in respect of another, we may consider the Sun, Earth and Planets as one system of bodies carried hither and thither by various motions among themselves, and the common center of gravity of all will either be quiescent, or move uniformly forward in a right line: In which case the whole system will likewise move uniformly forward in right lines. But this is an hypothesis hardly to be admitted. And therefore setting it aside, that common center will be quiescent. And from it the Sun is never far removed. The common center of gravity of the Sun and Jupiter falls on the surface of the Sun. And though all the Planets were placed towards the same parts from the Sun with Jupiter, the common center of the Sun and all of them would scarcely recede twice as far from the Sun's center. And therefore though the Sun, according to the various situation of the Planets, is variously agitated and always wandering to and fro

fro with a slow motion of libration, yet it never recedes one entire diameter of its own body from the quiescent center of the whole System. But from the weights of the Sun and Planets above determined, and the situation of all among themselves, their common center of gravity may be found, and this being given, the Sun's place to any supposed time may be obtained.

That the Planets nevertheless are revolved in ellipses, having their foci in the Sun; and by radii drawn to the Sun describe area's proportional to the times.

ABOUT the Sun thus librated the other Planets are revolved in elliptic orbits, and, by radii drawn to the Sun, describe area's nearly proportional to the times, as was explained before. If the Sun was quiescent, and the other planets did not act mutually one upon another, their orbits would be elliptic, and the area's exactly proportional to the times. But the actions of the Planets among themselves, compared with the actions of the Sun on the Planets, are of no moment, and produce no sensible errours. And those errours are less in revolutions about the Sun agitated in the manner but now described, than if those revolutions were made about the Sun quiescent, especially if the focus of every orbit is placed

in the common center of gravity of all the lower included Planets: *viz.* the focus of the orbit of Mercury, in the center of the Sun; the focus of the orbit of Venus, in the common center of gravity of Mercury and the Sun; the focus of the orbit of the Earth, in the common center of gravity of Venus, Mercury and the Sun; and so of the rest. And by this means the foci of the orbits of all the Planets except Saturn, will not be sensibly removed from the center of the Sun, nor will the focus of the orbit of Saturn recede sensibly from the common center of gravity of Jupiter and the Sun. And therefore astronomers are not far from the truth, when they reckon the Sun's center the common focus of all the planetary orbits. In Saturn it self, the errour thence arising does not exceed $1'. 45''$. And if its orbit, by placing the focus thereof in the common center of gravity of Jupiter and the Sun shall happen to agree better with the phænomena, from hence all that we have said will be farther confirmed.

Of the dimensions of the orbits, and of the motions of their aphelions and nodes.

If the Sun was quiescent and the Planets did not act one on another the aphelions and nodes of their orbits would likewise be quiescent. And the longer axes of their elliptic orbits would be as the cubic roots of the squares of their periodic times: And therefore from the given periodic times would be also given. But those times are to be measured not from the equinoctial points, which are moveable; but from the first Star of Aries. Put the semi-axe of the Earth's orbit 100000, and the semi-axes of the orbits of Saturn, Jupiter, Mars, Venus, and Mercury, from their periodic times will come out 953806, 520116, 152369, 72333, 38710 respectively. But from the Sun's motion every semi-axe is encreased by about one third of the distance of the Sun's center from the common center of gravity of the Sun and Planet. And from the actions of the exteriour Planets on the interiour, the periodic times of the interiour are something protracted though scarcely by any sensible quantity; and their aphelions are transferred by very slow motions in *consequentia*. And on the like account the periodic times of all, especially of the exteriour Planets, will be prolonged by

 the

the actions of the comets, if any such there are, without the orb of Saturn, and the aphelions of all will be thereby carried forwards in *consequentia*. But from the progress of the aphelions, the regress of the nodes follows. And if the plain of the Ecliptic is quiescent, the regress of the nodes will be to the progress of the aphelion in every orbit, as the regress of the nodes of the Moon's orbit to the progress of its apogæon nearly, that is, as about 10 to 21. But astronomical observations seem to confirm a very slow progress of the aphelions, and a regress of the nodes in respect of the fixed Stars. And hence it is probable, that there are comets in the regions beyond the Planets, which revolving in very eccentric orbs, quickly fly through their perihelion parts, and by an exceeding slow motion in their aphelions, spend almost their whole time in the regions beyond the Planets; as we shall afterwards explain more at large.

All the motions of the Moon that have hitherto been observed by astronomers, derived from the foregoing principles.

THE Planets thus revolved about the Sun, may at the same time carry others revolving about themselves as satellites or Moons. But from the action of the Sun, our Moon must move with greater velocity, and,

and, by a radius drawn to the Earth, deſcribe an area greater for the time; it muſt have its orbit leſs curve, and therefore approach nearer to the Earth, in the ſyzygies than in the quadratures, except in ſo far as the motion of eccentricity hinders thoſe effects. For the eccentricity is greateſt when the Moon's apogæon is in the ſyzygies, and leaſt when the ſame is in the quadratures; and hence it is that the perigeon-Moon is ſwifter and nearer to us, but the apogæon-Moon ſlower and farther from us, in the ſyzygies than in the quadratures. But further, the apogæon has a progreſſive, and the nodes a regreſſive motion, both unequable. For the apogæon is more ſwiftly progreſſive in its ſyzygies, more ſlowly regreſſive in its quadratures, and by the exceſs of its progreſs above its regreſs is yearly transferred in conſequentia: but the nodes are quieſcent in their ſyzygies, and moſt ſwiftly regreſſive in their quadratures. But further ſtill, the greateſt latitude of the Moon is greater in its quadratures than in its ſyzygies; and the mean motion ſwifter in the aphelion of the Earth than in its perihelion, More inequalities in the Moon's motion have not hitherto been taken notice of by aſtronomers. But

all these follow from our principles, and are known really to exist in the heavens. And this may be seen in that most ingenious, and, if I mistake not, of all the most accurate, hypothesis of Mr. *Horrox*, which Mr. *Flamsteed* has fitted to the heavens. But the astronomical hypotheses are to be corrected in the motion of the nodes. For the nodes admit the greatest equation or prosthaphæresis in their octants, and this inequality is most conspicuous, when the Moon is in the nodes, and therefore also in the octants. And hence it was that *Tycho* and others after him referred this inequality to the octants of the Moon, and made it menstrual. But the reasons by us adduced prove that it ought to be referred to the octants of the nodes, and to be made annual.

As also some other unequable motions that hitherto have not been observed.

BESIDE those inequalities taken notice of by astronomers, there are yet some others, by which the Moon's motions are so disturbed, that hitherto by no law could they be reduced to any certain regulation. For the velocities or horary motions of the apogee and nodes of the Moon, and their equations, as well as the difference betwixt the greatest

eſt eccentricity in the ſyzygies and the leaſt in the quadratures, and that inequality which we call the variation, in the progreſs of the year are augmented and diminiſhed in the triplicate ratio of the Sun's apparent diameter. Beſide that, the variation is mutable nearly in the duplicate ratio of the time between the quadratures. And all thoſe inequalities are ſomething greater in that part of the orbit, which reſpects the Sun, than in the oppoſite part, but by a difference that is ſcarcely or not at all perceptible.

And the diſtance of the Moon from the Earth to any given time.

By a computation which for brevities ſake I don't deſcribe, I alſo find, that the area which the Moon, by a radius drawn to the Earth, deſcribes in the ſeveral equal moments of time is nearly as the ſum of the number $237\frac{3}{10}$ and verſed ſine of the double diſtance of the Moon from the neareſt quadrature in a circle whoſe radius is unity; and therefore that the ſquare of the Moon's diſtance from the Earth, is as that ſum divided by the horary motion of the Moon. Thus it is when the variation in the octants is in its mean quantity. But if the variation is greater or leſs, that verſed ſine muſt be augmented or diminiſhed

ed in the ſame ratio. Let aſtronomers try how exactly the diſtances thus found will agree with the Moon's apparent diameters.

The motions of the ſatellites of Jupiter and Saturn, derived from the motions of our Moon.

FROM the motions of our Moon we may derive the motions of the Moons or Satellites of Jupiter and Saturn. For the mean motion of the nodes of the outmoſt ſatellite of Jupiter is to the mean motion of the nodes of our Moon in a proportion compounded of the duplicate proportion of the periodic time of the Earth about the Sun, to the periodic time of Jupiter about the Sun, and the ſimple proportion of the periodic time of the ſatellite about Jupiter to the periodic time of our Moon about the Earth, and therefore thoſe nodes, in the ſpace of an hundred years, are carried 8° 24′ backwards, or in antecedentia. The mean motions of the nodes of the inner ſatellites are to the (mean) motion of (the nodes of) the outmoſt as their periodic times to the periodic time of this, by the ſame corollary, and are thence given. And the motion of the apſis of every ſatellite in conſequentia is to the motion of its nodes in antecedentia, as the motion of

of the apogee of our Moon, to the motion of its nodes (by the same corollary) and is thence given. The variation of a satellite seen from Jupiter is to the variation of our Moon, in the same proportion, as the whole motions of their nodes respectively, during the times, in which the satellite and our Moon (after parting from) are revolved (again) to the Sun by the same corollary; and therefore in the outmost satellite the variation does not exceed 5''. 12'''. From the small quantity of those inequalities and the slowness of the motions, it happens that the motions of the satellites are found to be so regular, that the more modern astronomers either deny all motion to the nodes, or affirm them to be very slowly regressive.

That the Planets, in respect of the fixed Stars, are revolved by equable motions about their proper axes. And that (perhaps) those motions are the most fit for the equation of time.

WHILE the Planets are thus revolved in orbits about remote centers, in the mean time they make their several rotations about their proper axes; the Sun, in 26 days; Jupiter, in 9 h. 56'; Mars, in $24\frac{2}{3}$ h; Venus, in 23 h; and that in plains not much inclined to the plain of the Ecliptic, and according to the order of the

the ſigns, as aſtronomers determine from the ſpots or maculæ, that by turns preſent themſelves to our ſight in their bodies. And there is a like revolution of our Earth performed in 24 ^{h}. And thoſe motions are neither accelerated nor retarded by the actions of the centripetal forces. And therefore of all others they are the moſt equable and moſt fit for the menſuration of time. But thoſe revolutions are to be reckoned equable, not from their return to the Sun but to ſome fixed Star. For as the poſition of the Planets to the Sun is unequably varied, the revolutions of thoſe Planets from Sun to Sun are rendered unequable.

That the Moon likewiſe is revolved by a diurnal motion about its axe, and that its libration does thence ariſe.

In like manner is the Moon revolved about its axe by a motion moſt equable in reſpect of the fixed Stars, *viz.* in 27 ^{d}. 7 ^{h}. 43 $'$. that is, in the ſpace of a ſydereal month; ſo that this diurnal motion is equal to the mean motion of the Moon in its orbit. Upon which account, the ſame face of the Moon always reſpects the center about which this mean motion is performed, that is, the exteriour focus of the Moon's orbit nearly. And hence ariſes a deflexion

flexion of the Moon's face from the Earth, ſometimes towards the eaſt, at other times towards the weſt, according to the poſition of the focus which it reſpects; and this deflexion is equal to the equation of the Moon's orbit or to the difference betwixt its mean and true motions. And this is the Moon's libration in longitude. But it is likewiſe affected with a libration in latitude ariſing from the inclination of the Moon's axe to the plain of the orbit in which the Moon is revolved about the Earth. For that axe retains the ſame poſition to the fixed Stars nearly, and hence the poles preſent themſelves to our view by turns: as we may underſtand from the example of the motion of the Earth, whoſe poles, by reaſon of the inclination of its axe to the plain of the Ecliptic, are by turns illuminated by the Sun. To determine exactly the poſition of the Moon's axe to the fixed Stars, and the variation of this poſition, is a problem worthy of an aſtronomer.

Of the preceſſion of the equinoxes, and of the libratory motion of the axes of the Earth and Planets.

By reaſon of the diurnal revolutions of the Planets, the matter which they contain endeavours to recede from the axes of this motion; and hence

the fluid parts rising higher towards the equator than about the poles, would lay the solid parts about the equator under water, if those parts did not rise also. Upon which account the Planets are something thicker, about the equator than about the poles, and their equinoctial points thence become regressive, and their actions by a motion of nutation, twice in every revolution librate towards their ecliptics, and twice return again to their former inclination. And hence it is that Jupiter, viewed through very long telescopes, does not appear altogether round, but having its diameter that lies parallel to the Ecliptic, something longer than that which is drawn from north to south.

That the sea ought twice to flow, and twice to ebb every day; that the highest water must fall out in the third hour after the appulse of the luminaries to the meridian of the place.

And from the diurnal motion and the attractions of the Sun and Moon, our Sea ought twice to rise and twice to fall every day, as well Lunar as Solar, and the greatest hight of the water to happen before the sixth hour of either day, and after the twelfth hour preceding. By the slowness of the diurnal motion, the flood is retracted to the 12th hour, and

and by the force of the motion of reciprocation it is protracted and deferred till a time nearer to the sixth hour. But till that time is more certainly determined by the phænomena, choosing the middle between those extremes, why may we not conjecture the greatest height of the water to happen at the third hour? for thus the water will rise all that time in which the force of the luminaries to raise it is greater, and will fall all that time in which their force is less; viz. from the 9th to the 3d hour that force is greater, and from the 3d to the 9th it is less. The hours I reckon from the appulse of each luminary to the meridian of the place, as well under as above the horizon; and by the hours of the lunar day I understand the 24th parts of that time, which the Moon spends before it comes about again by its apparent diurnal motion to the meridian of the place, which it left the day before.

That the greatest tides happen in the syzygies of the luminaries, the least in their quadratures: and that at the third hour after the appulse of the Moon to the meridian of the place. But that out of the syzygies and quadratures those greatest and least tides deviate a little from that third hour towards the third hour after the appulse of the Sun to the meridian.

But the two motions which the two luminaries raise will not appear distinguished, but will make

a certain mixt motion. In the conjunction or opposition of the luminaries, their forces will be conjoined and bring on the greatest flood and ebb. In the quadratures the Sun will raise the waters which the Moon depresseth, and depress the waters which the Moon raiseth, and from the difference of their forces, the smallest of all tides will follow. And because (as experience tells us) the force of the Moon is greater than that of the Sun, the greatest height of the water will happen about the third lunar hour. Out of the syzygies and quadratures the greatest tide, which by the single force of the Moon ought to fall out at the third lunar hour, and by the single force of the Sun at the third solar hour, by the compound forces of both must fall out in an intermediate time that approaches nearer to the third hour of the Moon than to that of the Sun: And therefore while the Moon is passing from the syzygies to the quadratures, during which time the 3^d^ hour of the Sun precedes the 3^d^ of the Moon; and that by the greatest interval a little after the octants of the Moon; and by like intervals, the greatest tide will follow the 3^d^ lunar hour, while the Moon is passing from the quadratures to the syzygies.

BUT

But the effects of the luminaries depend upon their distances from the Earth. For when they are less distant, their effects are greater, and when more distant their effects are less, and that in the triplicate proportion of their apparent diameters. Therefore it is that the Sun in the winter time, being then in its perigee, has a greater effect, and makes the tides in the syzygies something greater, and those in the quadratures something less than in the summer season; and every month the Moon, while in the perigee, raiseth greater tides than at the distance of 15 days before or after, when it is in its apogee. Whence it comes to pass that two highest tides do not follow one the other, in two immediately succeeding syzygies.

That the tides are greatest when the luminaries are in their perigees.

The effect of either luminary doth likewise depend upon its declination or distance from the equator. For if the luminary was placed at the pole, it would constantly attract all the parts of the waters, without any intension or remission of its action, and could cause no reciprocation of motion. And therefore as the luminaries decline from the equator

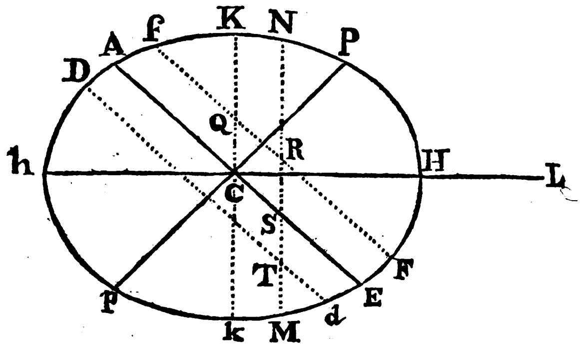
That the tides are greatest about the equinoxes.

 tor

tor towards either pole, they will by degrees lose their force, and on this account will excite lesser tides in the solstitial than in the equinoctial syzyiges. But in the solstitial quadratures, they will raise greater tides than in the quadratures about the equinoxes; because the force of the Moon, then situated in the equator, most exceeds the force of the Sun. Therefore the greatest tides fall out in those syzygies, and the least in those quadratures; which happen about the time of both equinoxes. And the greatest tide in the syzygies is always succeeded by the least tide in the quadratures, as we find by experience. But because the Sun is less distant from the Earth in winter than in summer, it comes to pass that the greatest and least tides more frequently appear before than after the vernal equinox; and more frequently after, than before the autumnal.

That out of the equator the tides are greater and less alternately.

MOREOVER the effects of the luminaries depend upon the latitudes of places. Let A p E P represent the Earth, on all sides covered with deep waters; C its center; P, p its poles; A E the equator; F any place without the equator; F f the parallel of the place; D d the correspondent parallel on the other



other ſide of the equator; L the place which the Moon poſſeſs'd three hours before; H the place of the Earth directly under it; *h* the oppoſite place; K, *k* the places at 90 degrees diſtance; C H, C *h* the greateſt heights of the Sea from the center of the Earth, and C K, C *k*, the leaſt heights: and if with the axes H *h*, K *k*, an ellipſe is deſcribed, and by the revolution of that ellipſe about its longer axe H *h*, a ſpheroid H P K *h p k* is formed; this ſpheroid will nearly repreſent the figure of the Sea; and C F, C *f*, C D, C *d*, will repreſent the figure of the Sea in the places F, *f*, D, *d*. But further, if in the ſaid revolution of the ellipſe any point N deſcribes the circle N M, cutting the parallels F *f*, D *d* in any places R, T, and the equa-

tor A E in S; C N will represent the height of the Sea in all those places, R, S, T, situated in this circle. Wherefore in the diurnal revolution of any place F, the greatest flood will be in F, at the 3d hour after the appulse of the Moon to the meridian above the horizon; and afterwards the greatest ebb in Q at the 3d hour after the setting of the Moon, and then the greatest flood in *f*, at the 3d hour after the appulse of the Moon to the meridian under the horizon, and lastly the greatest ebb in Q, at the 3d hour after the rising of the Moon; and the latter flood in *f*, will be less than the preceding flood in F. For the whole Sea is divided into two huge and hemispherical floods, one in the hemisphere K H *k* C, on the north-side, the other in the opposite hemisphere K H *k* C, which we may therefore call the northern and the southern floods. These floods being always opposite the one to the other, come by turns to the meridians of all places after the interval of 12 lunar hours. And seeing the northern countries partake more of the northern flood, and the southern countries more of the southern flood, thence arise tides alternately greater and less in all places without the equator, in which the luminaries rise and

and ſet. But the greater tide will happen when the Moon declines towards the vertex of the place, about the 3[d] hour after the appulſe of the Moon to the meridian above the horizon; and when the Moon changes its declination, that which was the greater tide will be changed into a leſſer, and the greateſt difference of the floods will fall out about the times of the ſolſtices, eſpecially if the aſcending node of the Moon is about the firſt of Aries. So the morning tides in winter exceed thoſe of the evening, and the evening tides exceed thoſe of the morning in ſummer; at *Plymouth* by the heighth of one foot, but at *Briſtol* by the heighth of 15 Inches, according to the obſervations of *Colepreſs* and *Sturmy*.

That by the conſervation of the impreſſed motion, the difference of the tides is diminiſhed: and that hence it may happen that the greateſt menſtrual tide will be the third after the ſyzygy.

BUT the motions which we have been deſcribing ſuffer ſome alteration from that force of reciprocation which the waters having once received, retain a little while by their *vis inſita*. Whence it comes to paſs that the tides may continue for ſome time, though the actions of the luminaries ſhould ceaſe. This power of retaining the impreſſed motion leſſens the difference of the alternate tides, and makes

those tides which immediately succeed after the syzygies greater, and those which follow next after the quadratures less. And hence it is that the alternate tides at *Plymoth* and *Bristol*, do not differ much more one from the other, than by the height of a foot or of 15 inches, and that the greatest tides of all at those ports are not the first but the third after the syzygies.

And besides all the motions are retarded in their passage through shallow channels, so that the greatest tides of all in some streights and mouths of rivers, are the fourth or even the fifth after the syzygies.

That the motions of the sea may be retarded by impediments in its channels.

It may also happen that the greatest tide may be the fourth or fifth after the syzygies, or fall out yet later, because the motions of the Sea are retarded in passing through shallow places towards the shores. For so the tide arrives at the western coast of *Ireland* at the third lunar hour, and an hour or two after, at the ports in the southern coast of the same Island, as also at the islands *Cassiterides* commonly *Sorlings*; then successively at *Falmouth*, *Plymouth*, *Portland*, the isle of *Wight*, *Winchester*, *Dover*, the mouth of the *Thames*,

Thames, and *London-Bridge*, ſpending 12 hours in this paſſage. But further the propagation of the tides may be obſtructed even by the channels of the ocean it ſelf, when they are not of depth enough; for the flood happens at the third lunar hour in the *Canary* Iſlands, and at all thoſe weſtern coaſts that lie towards the Atlantic ocean, as of *Ireland*, *France*, *Spain*, and all *Africa* to the cape of *Good-hope*, except in ſome ſhallow places, where it is impeded and falls out later; and in the ſtreights of *Gibraltar*, where by reaſon of a motion propagated from the Mediterranean Sea it flows ſooner. But paſſing from thoſe coaſts over the breadth of the ocean to the coaſts of *America*, the flood arrives firſt at the moſt eaſtern ſhoars of *Braſile*, about the fourth or fifth lunar hour, then at the mouth of the river of the *Amazons*, at the 6th hour, but at the neighbouring iſlands, at the 4th hour; afterwards at the iſlands of *Bermudas* at the 7th hour, and at port St. *Auguſtin* in *Florida* at $7\frac{1}{2}$. And therefore the tide is propagated through the ocean with a ſlower motion than it ſhould be according to the courſe of the Moon; and this retardation is very neceſſary, that the Sea at the ſame time may fall between *Braſile* and new *France*, and riſe at the

Canary Iſlands, and on the coaſts of *Europe* and *Africa*, and vice verſa. For the Sea cannot riſe in one place but by falling in another. And it is probable that the *Pacific* Sea is agitated by the ſame laws. For in the coaſts of *Chili* and *Peru*, the flood is ſaid to happen at the 3d lunar hour. But with what velocity it is thence propagated to the eaſtern coaſts of *Japan*, the *Philippine* and other iſlands adjacent to *China*, I have not yet learned.

That from the impediments of channels and ſhoars, various phænomena do ariſe, as that the ſea may flow but once every day.

FARTHER it may happen, that the tide may be propagated from the ocean through different channels towards the ſame port, and may paſs quicker through ſome channels than through others, in which caſe the ſame tide, divided into two or more ſucceeding one another, may compound new motions of different kinds. Let us ſuppoſe one tide to be divided into two equal tides; the former whereof precedes the other by the ſpace of 6 hours; and happens at the 3d or 27th hour of the appulſe of the Moon to the meridian of the port. If the Moon at the time of this appulſe to the meridian was in the equator, every ſix hours alternately there would ariſe equal floods, which meeting with as ma-

ny

ny equal ebbs, would ſo ballance one the other, that for that day the water would ſtagnate and remain quiet. If the Moon then declined from the equator, the tides in the ocean would be alternately greater and leſs as was ſaid. And from thence two greater and two leſſer tides would be alternately propagated towards that port. But the two greater floods would make the greateſt height of the waters to fall out in the middle time betwixt both, and the greater and leſſer floods would make the waters to riſe to a mean height in the middle time between them, and in the middle time between the two leſſer floods the waters would riſe to their leaſt heighth. Thus in the ſpace of 24 hours, the waters would come, not twice but, once only to their greateſt, and once only to their leaſt height; and their greateſt height, if the Moon declined towards the elevated Pole, would happen at the 6th or 3d hour after the appulſe of the Moon to the meridian, and when the Moon changed its declination this flood would be changed into an ebb.

Of all which we have an example in the port of *Batſham*, in the kingdom of *Tunquin*, in the latitude of 20°. 50′ north. In that port on the day which follows

follows after the paſſage of the Moon over the equator the waters ſtagnate: When the Moon declines to the north, they begin to flow and ebb, not twice as in other parts, but once only every day, and the flood happens at the ſetting, and the greateſt ebb at the riſeing of the Moon. This tide encreaſeth with the declination of the Moon, till the 7th or 8th day, then for the 7th or 8th day following, it decreaſeth at the ſame rate as it had increaſed before, and ceaſeth when the Moon changeth its declination. After which the flood is immediately changed into an ebb; and thenceforth the ebb happens at the ſetting, and the flood at the riſing of the Moon; till the Moon again changes its declination. There are two inlets from the ocean to this port; one more direct and ſhort between the iſland *Hainan* and the coaſt of *Quantung*, a province of *China*; the other round about between the ſame iſland and the coaſt of *Cochim*: And through the ſhorter paſſage the tide is ſooner propagated to *Batſham*.

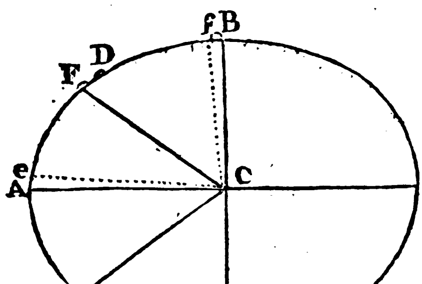
That the times of the tides within the channels of rivers are more unequal than in the ocean.

In the channels of rivers, the influx and reflux depends upon the current of the rivers, which obſtructs the ingreſs of the waters from the Sea, and promotes their egreſs to the Sea, making the ingreſs later and ſlower, and the egreſs ſooner and faſter. And hence it is, that the reflux is of longer duration than the influx, eſpecially far up the rivers, where the force of the Sea is leſs. So *Sturmy* tells us, that in the river *Avon* 3 miles below *Briſtol*, the water flows only 5 hours, but ebbs ſeven. And without doubt the difference is yet greater above *Briſtol*, as at *Careſham* or the *Bath*. This difference does likewiſe depend upon the quantity of the flux and reflux. For the more vehement motion of the Sea near the ſyzygies of the luminaries more eaſily overcoming the reſiſtance of the rivers, will make the ingreſs of the water to happen ſooner and to continue longer, and will therefore diminiſh this difference. But while the Moon is approaching to the ſyzygies, the rivers will be more plentifully filled, their currents being obſtructed by the greatneſs of the tides, and therefore will ſomething more retard the reflux of the Sea a little after, than a little before

before the ſyzygies. Upon which account the ſloweſt tides of all will not happen in the ſyzygies, but prevent them a little. And I obſerved above, that the tides before the ſyzygies were alſo retarded by the force of the Sun. And from both cauſes conjoined, the retardation of the tides will be both greater and ſooner before the ſyzygies. All which I find to be ſo, by the tide-tables which *Flamſteed* has compoſed from a great many obſervations.

That the tides are greater in greater and deeper Seas; greater on the ſhores of continents than of iſlands in the middle of the Sea; and yet greater in ſhallow bays that open with wide inlets to the Sea.

By the laws we have been deſcribing, the times of the tides are governed. But the greatneſs of the tides depends upon the greatneſs of the Seas. Let C repreſent the center of the Earth, E A D B the oval figure of the Sea, C A the long-



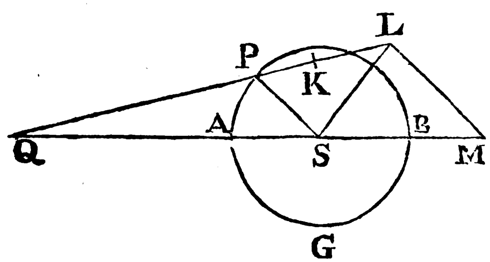
er

er ſemiaxe of this oval, C B the ſhorter inſiſting at right angles upon the former, D the middle point between A and B, and E C F or *e* C *f* the angle at the center of the Earth, ſubtended by the breadth of the Sea that terminates in the ſhores E, F; or *e*, *f*. Now ſuppoſing that the point A is in the middle between the points E, F, and the point D in the middle between the points *e*, *f*; if the difference of the heights C A, C B repreſent the quantity of the tide in a very deep Sea ſurrounding the whole Earth; the exceſs of the height C A above the height C E or C F, will repreſent the quantity of the tide in the middle of the Sea E F terminated by the ſhores E, F: and the exceſs of the height C *e* above the height C *f*, will nearly repreſent the quantity of the tide on the ſhores of the ſame Sea. Whence it appears that the tides are far leſs in the middle of the Sea, than at the ſhores; and that the tides at the ſhores are nearly as E F the breadth of the Sea, not exceeding a quadrantal arc. And hence it is that near the equator, where the Sea between *Africa* and *America* is narrow, the tides are far leſs than towards either ſide in the temperate Zones, where the Seas are extended wider, or on almoſt

almost all the shores of the *Pacific* Sea, as well towards *America* as towards *China*, and within as well as without the tropicks; and that in islands in the middle of the sea they scarce rise higher than 2 or 3 feet, but on the shores of great continents are 3 or 4 times greater and above, especially if the motions propagated from the ocean are by degrees contracted into a narrow space, and the water, to fill and empty the bays alternately, is forced to flow and ebb with great violence through shallow places; as at *Plymouth* and *Chepstow*-Bridge in *England*, at the mount of *St. Michael* and town of *Avranches* in *Normandy*, and at *Cambaja* and *Pegu* in the *East Indies*. In which places, the sea hurried in and out with great violence, sometimes lays the shores under water, sometimes leaves them dry for many miles. Nor is the force of the influx and efflux to be broke till it has raised or depressed the water to 40 or 50 feet and more. Thus also long and shallow streights that open to the sea with mouths wider and deeper than the rest of their channel, (such as those about *Britain*, and the *Magellanic Streights* at the eastern entry) will have a greater flood and ebb, or will more intend

intend and remit their courſe, and therefore will riſe higher and be depreſſed lower. On the coaſts of *South America*, 'tis ſaid that the *Pacific* Sea in its reflux ſometimes retreats two miles, and gets out of ſight of thoſe that ſtand on ſhoar. Whence in theſe places the floods will be alſo higher. But in deeper waters the velocity of influx and efflux is always leſs, and therefore the aſcent and deſcent is ſo too. Nor in ſuch places is the ocean known to aſcend to more than 6, 8, or 10 feet. The quantity of the aſcent I compute in the following manner.

Lel Q repreſent the Sun, S the



Earth, P the Moon, PAGB the Moon's orbit. In QP take QK equal to QS and QL to QK in the duplicate ratio of QK to QP. Parallel to PS draw LM; and ſuppoſing the mean quantity of

The force of the Sun to diſturb the motions of the moon computed from the foregoing principles.

of the circum-ſolar force directed towards the Earth to be repreſented by the diſtance QS or QK, QL will repreſent the quantity thereof directed towards the Moon. But that force is compounded of the parts QM, LM; of which the force LM, and that part of QM which is repreſented by SM do diſturb the motion of the Moon (as is evident from mathematical reaſoning). In ſo far as the Earth and Moon are revolved about their common center of gravity, the Earth will be liable to the action of the like forces. But we may refer the ſums as well of the forces as of the motions to the Moon, and repreſent the ſums of the forces, by the lines SM and ML, which are proportional to them. The force LM, in its mean quantity, is to the force by which the Moon may be revolved in an orbit about the Earth quieſcent at the diſtance PS, in the duplicate ratio of the Moon's periodic time about the Earth to the Earth's periodic time about the Sun, that is, in the duplicate ratio of $27^{d}. 7^{h}. 43'$. to $365^{d}. 6^{h}. 9'$. or as 1000 to 178725, or 1 to $178\frac{29}{40}$. The force by which the Moon may be revolved about the Earth in reſt, at the diſtance PS of $60\frac{1}{2}$ ſemi-diameters of the Earth, is

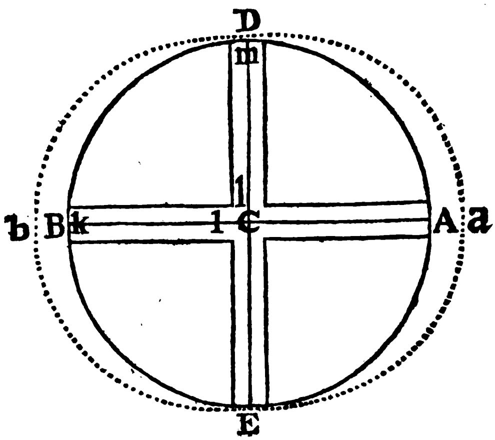
is to the force of gravity on the Earth's ſurface as 1 to $60\frac{4}{9} \times 60\frac{4}{9}$; and therefore the mean force M L is to the force of gravity as 1 to $60\frac{4}{9} \times 60\frac{4}{9} \times 178\frac{8}{11}$. Whence the force SM will be alſo given from the proportion of the lines S M, M L. And theſe are the forces of the Sun, by which the Moon's motions are diſturbed.

The force of the Sun to move the ſea computed.

If from the Moon's orbit we deſcend to the Earth's ſurface, thoſe forces will be diminiſhed in the ratio of the diſtances $60\frac{4}{9}$ and 1; and therefore the force L M will then become $60\frac{4}{9} \times 60\frac{4}{9} \times 178\frac{8}{11} \times 60\frac{4}{9}$ times leſs than the force of gravity. But this force acting equally every where upon the Earth, will ſcarcely effect any change on the motion of the Sea, and therefore may be neglected in the explication of that motion. The other force S M, in places where the Sun is vertical or in their nadir, is triple the quantity of the force M L, and therefore but $60\frac{4}{9} \times 60\frac{4}{9} \times 59\frac{19}{33}$ or 13184000 times leſs than the force of gravity.

SUPPOSE now ADBE to represent the spherical surface of the Earth, *a* D *b* E the surface of the water over-spreading it, C the center of both, A the place to which the Sun is vertical, B the place opposite; D, E, places at 90 degrees distance from the former; ACE *mlk* a

The height of the tide under the equator, arising from the force of the Sun, computed.



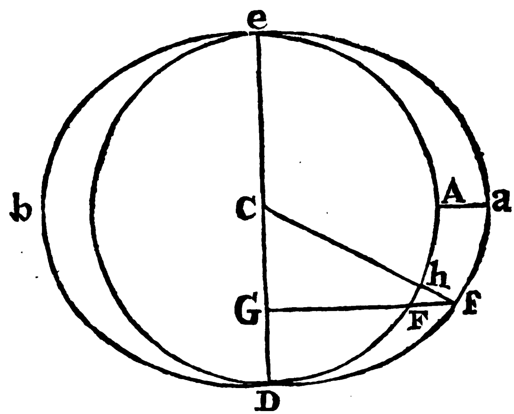
right angled cylindric canal passing thro' the Earth's center. The force SM in any place is as the distance of the place from the plain D E, on which a line from A to C insists at right angles, and therefore in the part of the canal which is represented by EC *l m*, is of no quantity; but in the other part AC *lk*, is as the gravity

vity at the several heights. For in descending towards the center of the Earth, gravity is every where as the height. And therefore the force SM drawing the water upwards will diminish its gravity in the leg AC*lk* of the canal in a given ratio; upon which account the water will ascend in this leg, till its defect of gravity is supplyed by its greater height, nor will it rest in an equilibrium, till its total gravity becomes equal to the total gravity in EC*lm* the other leg of the canal. Because the gravity of every particle is as its distance from the Earth's center, the weight of the whole water in either leg will increase in the duplicate ratio of the height; and therefore the height of the water in the leg AC*lk* will be to the height thereof in the leg C*lm*E in the subduplicate ratio of the number 13184001 to 13184000, or in the ratio of the number 26374001 to 26374000, and the height of the water in the leg EC*lm* to the difference of the heights, as 26374000 to 1. But the height in the leg EC*lm* is of 19615843 *Paris* feet, as has been lately found by the mensuration of the *French*. And therefore by the preceding analogy, the difference of the heights comes out 9 inches of the

the *Paris* foot; and the Sun's force will make the height of the Sea at A to exceed the height of the same at E by 9 inches. And though the water of the canal ACE*mlk* be supposed to be frozen into a hard and solid consistence, yet the heights thereof at A and E, and all other intermediate places would still remain the same.

The height of the tides under the parallels, arising from the Sun's force, computed.

LET A*a* represent that excess of height of nine inches at A, and *hf* the excess of height at any other place *h*; and upon DC let fall the perpendicular *f*G, meeting the globe of the Earth



in F. And because the distance of the Sun is so great that all the right lines drawn

drawn thereto may be considered as parallel, the force SM in any place F, will be to the same force in the place A, as the sine FG to the radius AC. And therefore since those forces tend to the Sun in the direction of parallel lines, they will generate the parallel heights F*f*, A*a* in the same ratio; and therefore the figure of the water D*faeb* will be a spheroid made by the revolution of an ellipse about its longer axe *ab*. And the perpendicular height *fh* will be to the oblique height F*f* as *f*G to *f*C, or as FG to AC: and therefore the height *fh* is to the height A*a* in the duplicate ratio of FG to AC, that is, in the ratio of the versed sine of double the angle DC*f* to double the radius, and is thence given. And hence to the several moments of the apparent revolution of the Sun about the Earth, we may infer the proportion of the ascent and descent of the waters at any given place under the equator, as well as of the diminution of that ascent and descent, whether arising from the latitude of places or from the Sun's declination; *viz.* That on account of the latitude of places, the ascent and descent of the sea is in all places diminished in the dupli-

cate ratio of the co-ſines of latitude; and on account of the Sun's declination, the aſcent and deſcent under the equator is diminiſhed in the duplicate ratio of the co-ſine of declination. And in places without the equator, the half ſum of the morning and evening aſcents (that is, the mean aſcent) is diminiſhed nearly in the ſame ratio.

The proportion of the tides under the equator, in the ſyzygies and quadratures, ariſing from the joint forces of both Sun and Moon.

LET S and L reſpectively repreſent the forces of the Sun and Moon placed in the equator, and at their mean diſtances from the Earth, r the radius, t and v the verſed ſines of double the complements of the Sun and Moon's declinations to any given time, D and E the mean apparent diameters of the Sun and Moon: and, ſuppoſing F and G to be their apparent diameters to that given time, their forces to raiſe the tides under the equator will be, in the ſyzygies, $\frac{vG^3}{2rE^3}L + \frac{tF^3}{2rD^3}S$, in the quadratures, $\frac{vG^3}{2rE^3}L - \frac{tF^3}{2rD^3}S$. And if the ſame ratio is likewiſe obſerved under the parallels, from obſervations accurately made in our northern climates,

climates, we may determine the proportion of the forces L and S; and then by means of this rule predict the quantities of the tides to every syzygy and quadrature.

The force of the Moon to excite tides, and the height of the water thence arising computed.

At the mouth of the river *Avon*, three miles below *Bristol*, in spring and autumn, the whole ascent of the water in the conjunction or opposition of the luminaries (by the observation of *Sturmy*) is about 45 feet, but in the quadratures only 25. Because the apparent diameters of the luminaries are not here determined, let us assume them in their mean quantities, as well as the Moon's declination in the equinoctial quadratures, in its mean quantity, that is, $23\frac{1}{2}°$; and the versed sine of double its complement will be 1682, supposing the radius to be 1000. But the declinations of the Sun in the equinoxes and of the Moon in the syzygies are of no quantity, and the versed sines of double the complements are each 2000. Whence those forces become L + S in the syzygies, and $\frac{1682}{2000}$ L — S in the quadratures, respectively proportional to the heights of the tides of 45 and 25

 feet,

feet, or of 9 and 5 paces. And therefore multiplying the extremes and the means we have $5\ L + 5\ S = \frac{15138}{2000}\ L$

$- 9\ S$, or $L = \frac{28000}{5138}\ S = 5\tfrac{5}{11}\ S$.

But further, I remember to have been told, that in summer the ascent of the sea in the syzygies, is to the ascent thereof in the quadratures as about 5 to 4. In the solstices themselves it is probable that the proportion may be something less, as about 6 to 5; whence it would follow that L is $= 5\tfrac{1}{3}\,S$. [For then the proportion is $\frac{1682}{2000}\ L + \frac{1682}{2000}$

$S : L - \frac{1682}{2000}\ S :: 6 : 5$.] Till we can more certainly determine the proportion from observation, let us assume $L = 5\tfrac{1}{3}\,S$; and since the heights of the tides are as the forces which excite them, and the force of the Sun is able to raise the tides to the height of nine inches, the Moon's force will be sufficient to raise the same to the height of four feet. And if we allow that this height may be doubled, or perhaps tripled by that force of reciprocation which

which we obſerve in the motion of the waters, and by which their motion once begun is kept up for ſome time, there will be force enough to generate all that quantity of tides, which we really find in the ocean.

That thoſe forces of the Sun and Moon are ſcarce ſenſible by any other effect beſide the tides which they raiſe in the ſea.

THUS we have ſeen that theſe forces are ſufficient to move the ſea. But, ſo far as I can obſerve, they will not be able to produce any other effect ſenſible on our Earth. For ſince the weight of one grain in 4000 is not ſenſible in the niceſt ballance; and the Sun's force to move the tides is 13187000 leſs than the force of gravity; and the ſum of the forces of both Moon and Sun, exceeding the Sun's force only in the ratio of $6\frac{1}{3}$ to 1, is ſtill above 2000000 times leſs than the force of gravity; it is evident that both forces together are 500 times leſs than what is required ſenſibly to increaſe or diminiſh the weight of any body in a ballance. And therefore they will not ſenſibly move any ſuſpended body; nor will they produce any ſenſible effect on pendulums, barometers, bodies ſwimming in ſtagnant water, or in the like ſtatical experiments. In the atmoſphere indeed

indeed they will excite ſuch a flux and reflux as they do in the ſea, but with ſo ſmall a motion that no ſenſible wind will be thence produced.

That the body of the Moon is about 6 times more denſe than the body of the Sun.

IF the effects of both Moon and Sun in raiſing the tides, as well as their apparent diameters were equal among themſelves, their abſolute forces would be as their magnitudes. But the effect of the Moon is to the effect of the Sun as about $5\frac{1}{3}$ to 1; and the Moon's diameter leſs than the Sun's in the ratio of $31\frac{1}{2}$ to $32\frac{1}{5}$ or of 45 to 46. Now the force of the Moon is to be increaſed in the ratio of the effect directly, and in the triplicate ratio of the diameter inverſely. Whence the force of the Moon compared with its magnitude will be to the force of the Sun compared with its magnitude in the ratio compounded of $5\frac{1}{3}$ to 1, and the triplicate of 45 to 46 inverſely, that is, in the ratio of about $5\frac{1}{10}$ to 1. And therefore the Moon in reſpect of the magnitude of its body, has an abſolute centripetal force greater than the Sun in reſpect of the magnitude of its body, in the ratio of $5\frac{1}{10}$ to 1, and is therefore more denſe in the ſame ratio.

IN

That the Moon is more denſe than the Earth in a ratio of about 3 to 2.

In the time of $27^{d}. 7^{h}. 43'$. in which the Moon makes its revolution about the Earth, a Planet may be revolved about the Sun at the diſtance of 18,954 diameters of the Sun from the Sun's center, ſuppoſing the mean apparent diameter of the Sun to be $32\frac{1}{5}'$. And in the ſame time the Moon may be revolved about the Earth at reſt, at the diſtance of 30 of the Earth's diameters. If in both caſes the number of diameters was the ſame, the abſolute circum-terreſtrial force would be to the abſolute circum-ſolar force as the magnitude of the Earth to the magnitude of the Sun. Becauſe the number of the Earth's diameters is greater in the ratio of 30 to 18,954, the body of the Earth will be leſs in the triplicate of that ratio, that is, in the ratio of $3\frac{28}{29}$ to 1. Wherefore the Earth's force, for the magnitude of its body, is to the Sun's force, for the magnitude of its body, as $3\frac{28}{29}$ to 1, and conſequently the Earth's denſity to the Sun's will be in the ſame ratio. Since then the Moon's denſity is to the Sun's denſity as $5\frac{7}{10}$ to 1, the Moon's denſity will be to the Earth's denſity as $5\frac{7}{10}$ to $3\frac{28}{29}$, or as 23 to 16. Wherefore ſince the Moon's magnitude is to the Earth's magnitude as

as about 1 to 41½, the Moon's absolute centripetal force will be to the Earth's absolute centripetal force as about 1 to 29, and the quantity of matter in the Moon to the quantity of matter in the Earth in the same ratio. And hence the common center of gravity of the Earth and Moon is more exactly determined than hitherto has been done. From the knowledge of which we may now infer the Moon's distance from the Earth with greater accuracy. But I would rather wait till the proportion of the bodies of the Moon and Earth one to the other is more exactly defined from the phænomena of the tides, hoping that in the mean time the circumference of the Earth may be measured from more distant stations than any body has yet employed for this purpose.

Of the distance of the fixed Stars.

THUS I have given an account of the system of the Planets. As to the fixed Stars, the smallness of their annual parallax proves them to be removed to immense distances from the system of the Planets. That this parallax is less than one minute is most certain; and from thence it follows that the distance of the fixed Stars is above 180 times greater than the distance

distance of Saturn from the Sun. Such as reckon the Earth one of the Planets and the Sun one of the fixed Stars, may remove the fixed Stars to yet greater distances by the following arguments. From the annual motion of the Earth there would happen an apparent transposition of the fixed Stars, one in respect of another, almost equal to their double parallax. But the greater and nearer Stars, in respect of the more remote which are only seen by the telescope, have not hitherto been observed to have the least motion. If we should suppose that motion to be but less than 20″. the distance of the nearer fixed Stars would exceed the mean distance of Saturn by above 1000 times. Again, the disc of Saturn, which is only 17″. or 18″. in diameter, receives but about $\frac{1}{2100000000}$ of the Sun's light. For so much less is that disc than the whole spherical surface of the orbit of Saturn. Now if we suppose Saturn to reflect about $\frac{1}{4}$ of this light, the whole light reflected from its illuminated hemisphere will be about $\frac{1}{4200000000}$ of the whole light emitted from the Sun's hemisphere.

ſphere. And therefore ſince light is rarefied in the duplicate ratio of the diſtance from the luminous body, if the Sun was $10000\sqrt{42}$ times more diſtant than Saturn, it would yet appear as lucid, as Saturn now does without its ring, that is, ſomething more lucid than a fixed Star of the firſt magnitude. Let us therefore ſuppoſe that the diſtance from which the Sun would ſhine as a fixed Star exceeds that of Saturn by about 10000 times, and its apparent diameter will be $7^{v}. 16^{vi}.$ and its parallax ariſing from the annual motion of the Earth $13''''$. And ſo great will be the diſtance, the apparent diameter, and the parallax, of the fixed Stars of the firſt magnitude in bulk and light equal to our Sun. Some may perhaps imagine that a great part of the light of the fixed Stars is intercepted and loſt in its paſſage through ſo vaſt ſpaces, and upon that account pretend to place the fixed Stars at nearer diſtances. But at this rate the remoter Stars could be ſcarce ſeen. Suppoſe for example that $\frac{3}{4}$ of the light periſh in its paſſage from the neareſt fixed Stars to us, and $\frac{3}{4}$ will twice periſh in its paſſage through a double ſpace, thrice thro' a triple, and ſo forth. And therefore the fixed Stars that are at a double diſtance will

will be 16 times more obſcure, *viz.* 4 times more obſcure on account of the diminiſhed apparent diameter; and again, 4 times more on account of the loſt light. And by the ſame argument, the fixed Stars at a triple diſtance, will be 9 × 4 × 4 or 144 times more obſcure, and thoſe at a quadruple diſtance will be 16 × 4 × 4 × 4 or 1024 times more obſcure. But ſo great a diminution of light is no ways conſiſtent with the phænomena and with that hypotheſis which places the fixed Stars at different diſtances.

That the Comets as often as they become viſible to us, are nearer than Jupiter. Proved from their parallax in longitude.

THE fixed Stars being therefore at ſuch vaſt diſtances from one another, can neither attract each other ſenſibly, nor be attracted by our Sun. But the Comets muſt unavoidably be acted on by the circum-ſolar force. For as the Comets were placed by aſtronomers above the Moon becauſe they were found to have no diurnal parallax; ſo their annual parallax is a convincing proof of their deſcending into the regions of the Planets. For all the Comets which move in a direct courſe, according to the order of the ſigns, about the end of their appearance become more than

than ordinarily ſlow, or retrograde, if the Earth is between them and the Sun; and more than ordinarily ſwift, if the Earth is approaching to a heliocentric oppoſition with them. Whereas on the other hand, thoſe which move againſt the order of the ſigns towards the end of their appearance, appear ſwifter than they ought to be if the Earth is between them and the Sun; and ſlower, and perhaps retrograde, if the Earth is in the other ſide of its orbit. This is occaſioned by the motion of the Earth in different ſituations. If the Earth go the ſame way with the Comet, with a ſwifter motion, the Comet becomes retrograde; if with a ſlower motion, the Comet becomes ſlower however; and if the Earth move the contrary way, it becomes ſwifter. And by collecting the differences between the ſlower and ſwifter motions, and the ſums of the more ſwift and of the retrograde motions, and comparing them with the ſituation and motion of the Earth from whence they ariſe, I found, by means of this parallax, that the diſtances of the Comets at the time they ceaſe to be viſible to the naked eye, is always leſs than the diſtance of Saturn; and generally even leſs than the diſtance of Jupiter.

THE ſame thing may be collected from the curvature of the way of the Comets. Theſe bodies go on nearly in great circles while their motion continues ſwift; but about the end of their courſe, when that part of their apparent motion, which ariſes from the parallax, bears a greater proportion to their whole apparent motion, they commonly deviate from thoſe circles; and when the Earth goes to one ſide, they deviate to the other. And this deflexion, becauſe of its correſponding with the motion of the Earth, muſt ariſe chiefly from the parallax; and the quantity thereof is ſo conſiderable, as by my computation, to place the diſappearing Comets a good deal lower than Jupiter. Whence it follows, that when they approach nearer to us in their perigees and perihelions, they often deſcend below the orbits of Mars and the inferiour Planets.

The ſame proved from their parallax in latitude.

MOREOVER, this nearneſs of the Comets is confirmed by the annual parallax of the orbit, in ſo far as the ſame is pretty nearly collected by the ſuppoſition that the Comets move uniformly in right lines. The method of collecting the diſtance

The ſame proved otherwiſe by the parallax.

of a Comet according to this hypothesis from four observations, (first attempted by *Kepler*, and perfected by Dr. *Wallis* and Sir *Christ. Wren*) is well known. And the Comets reduced to this regularity generally pass through the middle of the planetary region. So the Comets of the years 1607 and 1618 as their motions are defined by *Kepler*, passed between the Sun and the Earth; that of the year 1664 below the orbit of Mars; and that in 1680 below the orbit of Mercury, as its motion was defined by Sir *Christ. Wren* and others. By a like rectilinear hypothesis, *Hevelius* placed all the Comets about which we have any observations, below the orbit of Jupiter. 'Tis a false notion therefore, and contrary to astronomical calculation, which some have entertained, who from the regular motion of the Comets, either remove them into the regions of the fixed Stars, or deny the motion of the Earth; whereas their motions cannot be reduced to perfect regularity, unless we suppose them to pass through the regions near the Earth in motion. And these are the arguments drawn from the parallax, so far as it can be determined without an exact knowledge of the orbits and motions of the Comets.

THE

From the light of the Comets heads it is proved that they descend to the orbit of Saturn, Jupiter, and even of the Earth.

THE near approach of the Comets is further confirmed from the light of their heads. For the light of a celestial body, illuminated by the Sun, and receding to remote parts, is diminished in the quadruplicate proportion of the distance, to wit, in one duplicate proportion on account of the increase of the distance from the Sun; and in another duplicate proportion, on account of the decrease of the apparent diameter. Hence it may be inferred, that Saturn being at a double distance, and having its apparent diameter nearly half of that of Jupiter, must appear about 16 times more obscure; and that if its distance were 4 times greater, its light would be 256 times less; and therefore would be hardly perceivable to the naked eye. But now the Comets often equal Saturn's light, without exceeding him in their apparent diameters. So the Comet of the year 1678, according to Dr. *Hook*'s observations, equalled in brightness the light of a fixed Star of the first magnitude, and its head, or the Star in the middle of the coma, appeared, through a telescope of 15 foot, as lucid as Saturn near the horizon. But the diameter of the head was only 25''; that is, almost the same with

the diameter of a circle equal to Saturn and his ring. The coma or hair ſurrounding the head was about ten times as broad; namely, 4⅙ min. Again, the leaſt diameter of the hair of the Comet of the year 1682, obſerved by Mr. *Flamſteed* with a tube of 16 foot, and meaſured with the micrometer, was 2′. 0″. But the nucleus, or ſtar in the middle, ſcarce poſſeſſed the tenth part of this breadth, and was therefore only 11″. or 12″. broad. But the light and clearneſs of its head exceeded that of the year 1680, and was equal to that of the Stars of the firſt or ſecond magnitude. Moreover, the Comet of the year 1665 in *April*, as *Hevelius* informs us, exceeded almoſt all the fixed Stars in ſplendor; and even Saturn it ſelf, as being of a much more vivid colour. For this Comet was more lucid than that which appeared at the end of the foregoing year, and was compared to the Stars of the firſt magnitude. The diameter of the coma was about 6′. but the nucleus, compared with the Planets by means of a teleſcope, was plainly leſs than Jupiter, and was ſometimes thought leſs, ſometimes equal to, the body of Saturn within the ring. To this breadth add that of the ring, and the whole face of Saturn

turn will be twice as great as that of the Comet, with a light not at all more intense: and therefore the Comet was nearer to the Sun than Saturn. From the proportion of the nucleus to the whole head found by these observations, and from its breadth, which seldom exceeds 8′ or 12′, it appears that the Stars of the Comets are most commonly of the same apparent magnitude as the Planets; but that their light may be compared oftentimes with that of Saturn, and sometimes exceeds it. And hence 'tis certain that in their perihelia their distances can scarce be greater than that of Saturn. At twice that distance, the light would be four times less, which besides by its dim paleness would be as much inferiour to the light of Saturn, as the light of Saturn is to the splendour of Jupiter. But this difference would be easily observed. At a distance ten times greater their bodies must be greater than that of the Sun; but their light would be 100 times fainter than that of Saturn. And at distances still greater their bodies would far exceed the Sun: but being in such dark regions, they must be no longer visible. So impossible is it to place the Comets in the middle regions between the Sun and fixed Stars, account-

ing the Sun as one of the fixed Stars. For certainly they would receive no more light there from the Sun, than we do from the greateſt of the fixed Stars.

So far we have gone without conſidering that obſcuration which Comets ſuffer from that plenty of thick ſmoak which encompaſſeth their heads, and through which the heads always ſhew dull as through a cloud. For by how much the more a body is obſcured by this ſmoak, by ſo much the more near it muſt be allowed to come to the Sun, that it may vie with the Planets in the quantity of light which it reflects. Whence it is probable that the Comets deſcend far below the orbit of Saturn, as we proved before from their parallax. But above all, the thing is evinced from their tails, which muſt be owing either to the Sun's light reflected from a ſmoak ariſing from them and diſperſing it ſelf through the Æther, or to the lights of their own heads.

In the former caſe we muſt ſhorten the diſtance of the Comets, leaſt we be obliged to allow that the ſmoak ariſing from their heads is propagated through ſuch a vaſt extent of ſpace, and with ſuch a velocity of expanſion as will ſeem altogether incredible. In the latter

ter caſe the whole light of both head and tail muſt be aſcribed to the central nucleus. But then if we ſuppoſe all this light to be united and condenſed within the diſc of the nucleus, certainly the nucleus will by far exceed Jupiter it ſelf in ſplendour, eſpecially when it emits a very large and lucid tail. If therefore under a leſs apparent diameter it reflects more light, it muſt be much more illuminated by the Sun, and therefore much nearer to it. So the Comet that appeared *Dec.* 12. and 15. *O. S. Anno* 1679, at the time it emitted a very ſhining tail, whoſe ſplendour was equal to that of many Stars like Jupiter if their light were dilated and ſpread through ſo great a ſpace, was, as to the magnitude of its nucleus, leſs than Jupiter, (as Mr. *Flamſteed* obſerved) and therefore was much nearer to the Sun. Nay, it was even leſs than Mercury. For on the 17th of that month, when it was nearer to the Earth, it appeared to *Caſſini* through a teleſcope of 35 foot a little leſs than the globe of Saturn. On the 8th of this month, in the morning, Dr. *Halley* ſaw the tail, appearing broad and very ſhort, and as if it roſe from the body of the Sun it ſelf, at that time very near its riſing. Its form was like

that of an extraordinary bright cloud; nor did it disappear till the Sun it self began to be seen above the horizon. Its splendour therefore exceeded the light of the clouds till the Sun rose, and far surpassed that of all the Stars together; as yielding only to the immediate brightness of the Sun it self. Neither Mercury, nor Venus, nor the Moon it self are seen so near the rising Sun. Imagine all this dilated light collected together, and to be crouded into the orbit of the Comet's nucleus which was less than Mercury; by its splendour thus increased, becoming so much more conspicuous, it will vastly exceed Mercury, and therefore must be nearer to the Sun. On the 12th and 15th of the same month this tail extending it self over a much greater space, appeared more rare; but its light was still so vigorous as to become visible when the fixed Stars were hardly to be seen, and soon after to appear like a fiery beam shining in a wonderful manner. From its length, which was 40 or 50 deg. and its breadth of 2 deg. we may compute what the light of the whole must be.

The same proved from the extraordinary splendour of their tails when they are near the Sun.

This near approach of the Comets to the Sun is confirmed from the situation they are seen in when their tails appear most resplendent. For when the head passes by the Sun and lies hid under the solar rays, very bright and shining tails like fiery beams are said to issue from the horizon; but afterwards when the head begins to appear, and is got farther from the Sun, that splendour always decreases and turns by degrees into a paleness, like to that of the milky way, but much more sensible at first; after that vanishing gradually. Such was that most resplendent Comet described by *Aristotle*, *Lib.* 1. *Meteor*. 6. 'The 'head thereof could not be seen because 'it set before the Sun, or at least was hid 'under the Sun's rays; but the next day 'it was seen as well as might be; for ha-'ving left the Sun but a very little way it 'set immediately after it. And the scat-'tered light of the head obscured by the 'too great splendour (of the tail) did not 'yet appear. But afterwards (says *Ari-'stotle*) when the splendour of the tail 'was now diminished, (the head of) the 'Comet recovered its native brightness. 'And the splendour of its tail reached 'now to a third part of the heavens' (that

is

is to 60°.) 'It appeared in the winter 'season, and rising to *Orion*'s Girdle, 'there vanished away.' Two Comets of the same kind are described by *Justin*, *lib*. 37. which according to his account 'shined so bright that the whole heaven 'seemed to be on fire; and by their great-'ness filled up a fourth part of the hea-'vens, and by their splendour exceeded 'that of the Sun.' By which last words a near position of these bright Comets and the rising or setting Sun is intimated. We may add to these the Comet of the year 1101 or 1106, 'the star of which 'was small and obscure (like that of 1680) 'but the splendour arising from it ex-'tremely bright, reaching like a fiery 'beam to the East and North,' as *Heve-lius* has it from *Simeon* the monk of *Durham*. It appeared at the beginning of *February* about the evening in the south-west. From this and from the si-tuation of the tail we may infer that the head was near the Sun. *Matthew Paris* says, 'it was about one cubit from the 'Sun; from the third' [or rather the sixth] 'to the ninth hour sending out a 'long stream of light.' The Comet of 1264 in *July*, or about the solstice, pre-ceded the rising Sun, sending out its beams with a great light towards the west

weſt as far as the middle of the heavens. And at the beginning it aſcended a little above the horizon; but as the Sun went forwards it retired every day farther from the horizon, till it paſſed by the very middle of the heavens. It is ſaid to have been at the beginning large and bright, having a large coma, which decayed from day to day. It is deſcribed in *Append. Matth. Pariſ. Hiſt. Ang.* after this manner. *An. Chriſti* 1265. 'there appeared a Comet ſo wonderful, 'that none then living had ever ſeen the 'like. For riſing from the eaſt with a 'great brightneſs, it extended it ſelf with 'a great light as far as the middle of the 'hemiſphere towards the weſt.' The Latin original being ſomewhat barbarous and obſcure, it is here ſubjoined. *Ab oriente enim cum magno fulgore ſurgens, uſque ad medium hemiſphærii verſus occidentem, omnia perlucide pertrahebat.* In the year 1401 or 1402, the Sun being got below the horizon, there appeared in the weſt a bright and ſhining Comet ſending out a tail upwards in ſplendour like a flame of fire, and in form like a ſpear, darting its rays from weſt to eaſt. When the Sun was ſunk below the horizon, by the luſtre of its own rays it enlightned all the borders of

of the Earth, not permitting the other Stars to shew their light, or the shades of night to darken the air, because its light exceeded that of the others, and extended it self to the upper part of the heavens, &c. *Hist. Byzant. Duc. Mich. Nepot.* From the situation of the tail of this Comet, and the time of its first appearance, we may infer that the head was then near the Sun, and went farther from him every day. For that Comet continued three months. In the year 1527, *Aug.* 11. about four in the morning, there was seen almost throughout *Europe*, a terrible Comet in *Leo*, which continued flaming an hour and a quarter every day. It rose from the east, and ascended to the south and west to a prodigious length. It was most conspicuous to the north, and its cloud (that is its tail) was very terrible; having, according to the fancies of the vulgar, the form of an arm a little bent, holding a sword of a vast magnitude. In the year 1618, in the end of *November*, there began a rumour, that there appeared about Sun rising a bright beam, which was the tail of a Comet, whose head was as yet concealed within the brightness of the solar rays. On *Nov.* 24. and from that time the comet it self appeared with

a

a bright light, its head and tail being extremely reſplendent. The length of the tail, which was at firſt 20 or 30 deg. increaſed till *December* 9. when it aroſe to 75 deg. but with a light much more faint and dilute than at the beginning. In the year 1668, *March* 5. *N. S.* P. *Valent. Eſtancius* being in *Braſile*, ſaw a comet near the horizon in the ſouth-weſt. Its head was ſmall, and ſcarce diſcernible, but its tail extremely bright and refulgent, ſo that the reflexion of it from the ſea was eaſily ſeen by thoſe who ſtood upon the ſhore. This great ſplendour laſted but three days, decreaſing very remarkably from that time. The tail at the beginning extended it ſelf from weſt to ſouth, and in a ſituation almoſt parallel to the horizon, appearing like a ſhining beam 23 deg. in length. Afterwards the light decreaſing, its magnitude increaſed till the Comet ceaſed to be viſible. So that *Caſſini* at *Bologna* ſaw it (*Mar*. 10. 11. 12.) riſing from the horizon 32 deg. in length. In *Portugal* it is ſaid to have taken up a fourth part of the heavens, (that is, 45 deg.) extending it ſelf from weſt to eaſt with a notable brightneſs; though the whole of it was not ſeen, becauſe the head in this part of the world always

always lay hid below the horizon. From the increase of the tail it is plain that the head receded from the Sun, and was nearest to it at the beginning when the tail appeared brightest.

To all these we may add the comet of 1680, whose wonderful splendour at the conjunction of the head with the Sun was above described. But so great a splendour argues the comets of this kind to have really passed near the fountain of light; especially since the tails never shine so much in their opposition to the Sun; nor do we read that fiery beams have ever appeared there.

The same proved from the light of their heads, as being greater *cæteris paribus* when they come near to the Sun.

LASTLY, the same thing is inferred from the light of the heads increasing in the recess of the comets from the Earth towards the Sun, and decreasing in their return from the Sun towards the Earth. For so the last comet of the year 1665, (by the observation of *Hevelius*) from the time that it was first seen was always losing of its apparent motion, and therefore had already passed its perigee, yet the splendour of its head was daily increasing, till being hid by the Sun's rays, the comet ceased to appear. The Comet of the year 1683, (by

(by the obſervation of the ſame *Hevelius*) about the end of *July*, when it firſt appeared, moved at a very ſlow rate, advancing only about 40 or 45 minutes in its orbit in a day's time. But from that time its diurnal motion was continually upon the increaſe till *September* 4. when it aroſe to about 5 degrees. And therefore in all this interval of time the Comet was approaching to the Earth. Which is likewiſe proved from the diameter of its head meaſured with a micrometer. For *Auguſt* the 6th *Hevelius* found it only $6'. 5''$. including the coma; which *September* 2. he obſerved $9'. 7''$. And therefore its head appeared far leſs about the beginning than towards the end of its motion, though about the beginning, becauſe nearer to the Sun, it appeared far more lucid than towards the end, as the ſame *Hevelius* declares. Wherefore in all this interval of time, on account of its receſs from the Sun, it decreaſed in ſplendour, notwithſtanding its acceſs towards the Earth. The comet of the year 1618, about the middle of *December*, and that of the year 1680, about the end of the ſame month, did both move with their greateſt velocity, and were therefore then in their perigees. But the greateſt ſplen-

ſplendour of their heads was ſeen two weeks before when they had juſt got clear of the Sun's rays. And the greateſt ſplendour of their tails a little more early, when yet nearer to the Sun. The head of the former Comet, according to the obſervations of *Cyſatus*, *Dec.* 1. appeared greater than the Stars of the firſt magnitude, but in the ſplendour and brightneſs of its light, a great deal. *Jan.* 7. *Kepler* being uncertain about the head, left off obſerving. *Dec.* 12. the head of the laſt Comet was ſeen and obſerved by *Flamſteed* at the diſtance of 9 degrees from the Sun; which a Star of the third magnitude could hardly have been. *December* 15 and 17, the ſame appeared like a Star of the third magnitude, its ſplendour being diminiſhed by the bright clouds near the ſetting Sun. *Dec.* 26. when it moved with the greateſt ſwiftneſs, and was almoſt in its perigee, it was inferiour to *Os Pegaſi*, a Star of the third magnitude. *Jan.* 3. it appeared like a Star of the fourth. *Jan.* 9. like a Star of the fifth. *Jan.* 13. it diſappeared by reaſon of the brightneſs of the Moon which was then in its increaſe. *Jan.* 25. it was ſcarce equal to the Stars of the ſeventh magnitude. If we take equal times on each hand of the perigee, the

the heads placed at remote diſtances would have ſhined equally before and after, becauſe of their equal diſtances from the Earth. That in one caſe they ſhined very bright, and in the other vaniſhed, is to be aſcribed to the nearneſs of the Sun in the firſt caſe, and his diſtance in the other. And from the great difference of the light in theſe two caſes, we infer its great nearneſs in the firſt of them. For the light of the Comets uſes to be regular, and to appear greateſt when their heads move the ſwifteſt, and are therefore in their perigees; excepting in ſo far as it is increaſed by their nearneſs to the Sun.

The ſame confirmed by the great number of Comets ſeen in the region of the Sun.

FROM theſe things I at laſt diſcovered why the Comets frequent ſo much the region of the Sun. If they were to be ſeen in the regions a great way beyond Saturn, they muſt appear oftner in thoſe parts of the heavens that are oppoſite to the Sun. For thoſe which are in that ſituation would be nearer to the Earth; and the interpoſition of the Sun would obſcure the others. But looking over the hiſtory of Comets, I find that four or five times more have been ſeen in the hemiſphere toward the Sun, than in the

opposite hemisphere; besides, without doubt, not a few which have been hid by the light of the Sun. For Comets descending into our parts neither emit tails, nor are so well illuminated by the Sun as to discover themselves to our naked eyes, till they are come nearer to us than Jupiter. But the far greater part of that sphærical space, which is described about the Sun with so small an interval, lies on that side of the Earth which regards the Sun, and the Comets in that greater part are more strongly illuminated as being for the most part nearer to the Sun. Besides, from the remarkable eccentricity of their orbits it comes to pass that their lower apsides are much nearer to the Sun than if their revolutions were performed in circles concentric to the Sun.

This also confirmed, by the greater magnitude and splendour of the tails after the conjunction of the heads with the Sun, than before.

HENCE also we understand why the tails of the Comets, while their heads are descending towards the Sun, always appear short and rare, and are seldom said to have exceeded 15 or 20 deg. in length; but in the recess of the heads from the Sun often shine like fiery beams, and soon after reach to 40, 50, 60, 70 deg. in length, or more. This great splendour

ſplendour and length of the tails ariſes from the heat which the Sun communicates to the Comet as it paſſes near it. And thence I think it may be concluded that all the Comets that have had ſuch tails have paſſed very near the Sun.

That the tails ariſe from the atmoſpheres of the Comets.

HENCE alſo we may collect that the tails ariſe from the atmoſpheres of the heads. But we have had three ſeveral opinions about the tails of Comets. For ſome will have it, that they are nothing elſe but the beams of the Sun's light tranſmitted through the Comets heads, which they ſuppoſe to be tranſparent: others, that they proceed from the refraction which light ſuffers in paſſing from the Comets head to the earth: and laſtly, others, that they are a ſort of clouds or vapour conſtantly riſing from the Comets heads, and tending towards the parts oppoſite to the Sun. The firſt is the opinion of ſuch as are yet unacquainted with opticks. For the beams of the Sun are not ſeen in a darkned room, but in conſequence of the light that is reflected from them, by the little particles of duſt and ſmoak which are always flying about in the air. And hence it is, that in air impregnated with thick

thick ſmoak they appear with greater brightneſs, and are more faintly and more difficultly ſeen in a finer air. But in the heavens, where there is no matter to reflect the light they are not to be ſeen at all. Light is not ſeen as it is in the beams, but as it is thence reflected to our eyes. For viſion is not made but by rays falling upon the eyes; and therefore there muſt be ſome reflecting matter in thoſe parts where the tails of Comets are ſeen; and ſo the argument turns upon the third opinion. For that reflecting matter can be no where found but in the place of the tail, becauſe otherwiſe, ſince all the celeſtial ſpaces are equally illuminated by the Sun's light, no part of the heavens could appear with more ſplendour than another. The ſecond opinion is liable to many difficulties. The tails of Comets are never ſeen variegated with thoſe colours, which ever uſe to be inſeparable from refraction. And the diſtinct tranſmiſſion of the light of the fixed Stars and Planets to us, is a demonſtration that the æther or celeſtial medium is not endowed with any refractive power. For as to what is alledged that the fixed Stars have been ſometimes ſeen by the *Egyptians*, environed with a coma or capillitium,

litium, becauſe that has but rarely happened, it is rather to be aſcribed to a caſual refraction of clouds, as well as the radiation and ſcintillation of the fixed Stars to the refractions both of the eyes and air. For upon applying a teleſcope to the eye, thoſe radiations and ſcintillations immediately diſappear. By the tremulous agitation of the air and aſcending vapours it happens that the rays of light are alternately turned aſide from the narrow ſpace of the pupil of the eye; but no ſuch thing can have place in the much wider aperture of the object-glaſs of a teleſcope. And hence it is, that a ſcintillation is occaſioned in the former caſe, which ceaſes in the latter. And this ceſſation in the latter caſe is a demonſtration of the regular tranſmiſſion of light through the heavens without any ſenſible refraction. But to obviate an objection that may be made from the appearing of no tail in ſuch Comets as ſhine but with a faint light, as if the ſecondary rays were then too weak to affect the eyes, and for this reaſon it is that the tails of the fixed Stars do not appear; we are to conſider, that by the means of teleſcopes the light of the fixed Stars may be augmented above an hundred fold, and yet

no tails are seen That the light of the Planets is yet more copious without any tail, but that Comets are seen sometimes with huge tails, when the light of their heads is but faint and dull. For so it happened in the Comet of the year 1680, when in the month of *December* it was scarcely equal in light to the Stars of the second magnitude, and yet emitted a notable tail, extending to the length of 40°, 50°, 60°, or 70°, and upwards; and afterwards on the 27th and 28th of *January*, when the head appeared but as a Star of the 7th magnitude; but the tail, (as was said above) with a light that was sensible enough though faint, was stretched out to 6 or 7 degrees in length, and with a languishing light, that was more difficultly seen, even to 12°. and upwards. But on the 9th and 10th of *February*, when to the naked eye the head appeared no more through a telescope, I viewed the tail of 2°. in length. But further, if the tail was owing to the refraction of the celestial matter, and did deviate from the opposition of the Sun, according as the figure of the heavens requires; that deviation, in the same places of the heavens, should be always directed towards the same parts. But the Comet of

of the year 1680 *Dec.* 28 d. 8 ½ h. P. M. at *London* was ſeen in ♓ 8°. 41′ with latitude north 28°. 6′. while the Sun was in ♑ 18°. 26′. And the Comet of the year 1577, *December* 29, was in ♓ 8°. 41′. with latitude north 28°. 40′. and the Sun as before in about 18°. 26′. In both caſes the ſituation of the Earth was the ſame, and the Comet appeared in the ſame place of the heavens; yet in the former caſe the tail of the Comet (as well by my obſervations as by the obſervations of others) deviated from the oppoſition of the Sun towards the north by an angle of 4½ degrees, whereas in the latter there was (according to the obſervation of *Tycho*) a deviation of 21 degrees towards the ſouth. The refraction therefore of the heavens being thus diſproved, it remains that the phænomena of the tails of Comets muſt be derived from ſome reflecting matter. That vapours ſufficient to fill ſuch immenſe ſpaces may ariſe from the Comets atmoſpheres, may be eaſily underſtood by what follows.

That the air and vapour in the celestial spaces is of an immense rarity: and that a small quantity of vapour may be sufficient to explain all the phænomena of the tails of Comets.

It is well known that the air near the surface of our Earth possesses a space about 1200 times greater than water of the same weight. And therefore a cylindric column of air 1200 feet high, is of equal weight with a cylinder of water of the same breadth and but one foot high. But a cylinder of air reaching to the top of the atmosphere is of equal weight with a cylinder of water about 33 foot high; and therefore if from the whole cylinder of air, the lower part of 1200 foot high is taken away, the remaining upper part will be of equal weight with a cylinder of water 32 foot high Wherefore at the height of 1200 feet or two furlongs, the weight of the incumbent air is less, and consequently the rarity of the compressed air greater than near the surface of the Earth in the ratio of 33 to 32. And having this ratio, we may compute the rarity of the air in all places whatsoever, supposing the expansion thereof to be reciprocally proportional to its compression; and this proportion has been proved by the experiments of *Hook* and others. The result of the computation I have set down in the following table, in the first column of

of which you have the height of the air in miles whereof 4000 make a ſemi-diameter of the Earth; in the ſecond the compreſſion of the air, or the incumbent weight; in the third its rarity or expanſion, ſuppoſing gravity to decreaſe in the duplicate ratio of the diſtances from the Earth's center. And the Latin numeral characters are here uſed for certain numbers of ciphers, as 0,xvii 1224 for 0,000000000000000001224, and 26956xv for 26956000000000000000.

AIR'S

Height.	Compreſſion.	Expanſion.
0	33	1
5	17,8515	1,8486
10	9,6717	3,4151
20	2,852	11,571
40	0,2525	136,83
400	0,xvii 1224	26956 xv
4000	0,cv 4465	73907 cii
40000	0,cxcii 1628	20263 clxxxix
400000	0,ccx 7895	41798 ccvii
4000000	0,ccxii 9878	33414 ccix
Infinite.	0,ccxii 6041	54622 ccix

But from this table it appears that the air, in proceeding upwards, is rarified in ſuch manner, that a ſphere of that air which

which is nearest to the Earth of but one inch in diameter, if dilated with an equal rarefaction with that of the air at the height of ten semidiameters of the Earth, would fill up more space than is contained in the whole heavens on this side the fixed Stars, according to the preceding computation of their distance. And though by reason of the far greater thickness of the atmospheres of Comets, and the great quantity of the circum-solar centripetal force it may happen that the air in the celestial spaces, and in the tails of Comets, is not so vastly rarified; yet from this computation it is plain, that a very small quantity of air and vapour is abundantly sufficient to produce all the appearances of the tails of Comets. For that they are indeed of a very notable rarity appears from the shining of the Stars through them. The atmosphere of the Earth, illuminated by the Sun's light, though but of a few miles in thickness, obscures and extinguishes the light, not only of all the Stars, but even of the Moon it self; whereas the smallest Stars are seen to shine through the immense thickness of the tails of Comets, likewise illuminated by the Sun, without the least diminution of their splendour.

KEPLER

KEPLER aſcribes the aſcent of the tails of Comets to the atmoſpheres of their heads, and their direction towards the parts oppoſite to the Sun, to the action of the rays of light, carrying along with them the matter of the Comets tails. And without any great incongruity we may ſuppoſe that, in ſo free ſpaces, ſo fine a matter as that of the Ether may yield to the action of the rays of the Sun's light, though thoſe rays are not able ſenſibly to move the groſs ſubſtances in our parts, which are clogged with ſo palpable a reſiſtance. Another author thinks that there may be a ſort of particles of matter endowed with a principle of levity as well as others are with a power of gravity; that the matter of the tails of Comets may be of the former ſort, and that its aſcent from the Sun may be owing to its levity. But conſidering the gravity of terreſtial bodies is as the matter of the bodies, and therefore can be neither more nor leſs in the ſame quantity of matter, I am inclined to believe that this aſcent may rather proceed from the rarefaction of the matter of the Comets tails. The aſcent of ſmoak in a chimney is owing to the impulſe of the air, with which it

After what manner the tails of Comets may ariſe from the atmoſpheres of their heads.

it is entangled. The air rarified by heat aſcends, becauſe its ſpecifick gravity is diminiſhed, and in its aſcent carries along with it the ſmoak with which it is ingaged. And why may not the tail of a Comet riſe from the Sun after the ſame manner? For the Sun's rays do not act any way upon the mediums which they pervade, but by reflection and refraction. And thoſe reflecting particles heated by this action, heat the matter of the Ether which is involved with them. That matter is rarefied by the heat which it acquires, and becauſe by this rarefaction the ſpecifick gravity, with which it tended towards the Sun before, is diminiſhed, it will aſcend therefrom like a ſtream, and carry along with it the reflecting particles of which the tail of the comet is compoſed; the impulſe of the Sun's light, as we have ſaid, promoting the aſcent.

That the tails do indeed ariſe from thoſe atmoſpheres, proved from ſeveral of their phænomena.

BUT that the tails of Comets do ariſe from their heads and tend towards the parts oppoſite to the Sun, is further confirmed from the laws which the tails obſerve. For lying in the plains of the Comets orbit which paſs through the Sun, they conſtantly deviate from the

the opposition of the Sun, towards the parts which the Comets heads in their progress along those orbits have left. And to a spectator placed in those plains, they appear in the parts directly opposite to the Sun. But as the spectator recedes from those plains, their deviation begins to appear, and daily becomes greater. And the deviation *cæteris paribus* appears less, when the tail is more oblique to the orbit of the Comet, as well as when the head of the Comet approaches nearer to the Sun; especially if the angle of deviation is estimated near the head of the Comet. Farther, the tails which have no deviation appear streight, but the tails which deviate are likewise bended into a certain curvature. And this curvature is greater when the deviation is greater; and is more sensible when the tail *cæteris paribus* is longer: for in the shorter tails the curvature is hardly to be perceived. And the angle of deviation is less near the Comet's head, but greater towards the other end of the tail, and that because the lower side of the tail regards the parts from which the deviation is made, and which lie in a right line, drawn out infinitely from the Sun through the Comet's head. And the tails that are longer and broader, and shine

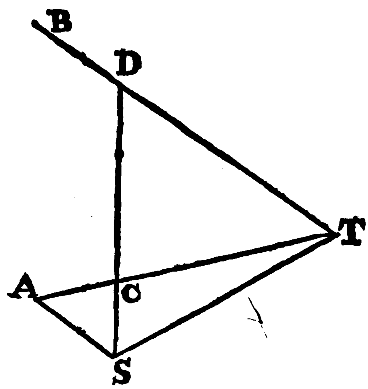
ſhine with a ſtronger light, appear more reſplendent and more exactly defined on the convex than on the concave ſide. Upon which accounts it is plain that the phænomena of the tails of Comets depend upon the motions of their heads, and by no means upon the places of the heavens in which their heads are ſeen; and that therefore the tails of the Comets do not proceed from the refraction of the heavens, but from their own heads, which furniſh the matter that forms the tail. For as in our air the ſmoak of a heated body aſcends either perpendicularly if the body is at reſt, or obliquely, if the body is moved obliquely; ſo in the heavens where all the bodies gravitate towards the Sun, ſmoak and vapour muſt (as we have already ſaid) aſcend from the Sun, and either riſe perpendicularly if the ſmoaking body is at reſt, or obliquely if the body, in the progreſs of its motion, is always leaving thoſe places from which the upper or higher parts of the vapours had riſen before. And that obliquity will be leſs, where the vapour aſcends with more velocity, to wit, near the ſmoaking body when that is near the Sun. For there the force of the Sun by which the vapour aſcends is ſtronger. But becauſe the obliquity is varied, the column of

of vapour will be incurvated; and because the vapour in the preceding ſide is ſomething more recent, that is, has aſcended ſomething more lately from the body, it will therefore be ſomething more denſe on that ſide, and muſt on that account reflect more light as well as be better defined; the vapour on the other ſide languiſhing by degrees and vaniſhing out of ſight.

That Comets do ſometimes deſcend below the orbit of Mercury, proved from their tails.

But it is none of our preſent buſineſs to explain the cauſes of the appearances of nature. Let thoſe things which we have laſt ſaid be true or falſe, we have at leaſt made out in the preceding diſcourſe, that the rays of light are directly propagated from the tails of Comets in right lines through the heavens, in which thoſe tails appear to the ſpectators where-ever placed; and conſequently the tails muſt aſcend from the heads of the comets towards the parts oppoſite to the Sun. And from this principle we may determine the limits of their diſtances in manner following. Let S repreſent the Sun, T the Earth, S T A the elongation of a Comet from the Sun, and ATB the apparent length of its tail. And becauſe the light is propagated from the extremity of the tail

tail in the direction of the right line TB, that extremity must lie somewhere in the line TB. Suppose it in D, and join DS cutting TA in C. Then because the tail is always stretched out towards the parts nearly opposite to the Sun; and therefore the Sun, the head of the Comet, and the extremity of the tail lie in a right line, the Comet's head will be found in C. Parallel to TB draw SA,



meeting the line TA in A, and the Comet's head C must necessarily be found between T and A; because the extremity of the tail lies somewhere in the infinite line TB, and all the lines SD which can possibly be drawn from the point S to the line TB, must cut the line TA somewhere between T and A. Where-

Wherefore the diſtance of the Comet from the Earth cannot exceed the interval TA, nor its diſtance from the Sun the interval SA beyond, or ST on this ſide, the Sun. For inſtance, the elongation of the Comet of 1680 from the Sun *Dec.* 12. was 9°. and the length of its tail 35°. at leaſt. If therefore a triangle TSA is made whoſe angle T is equal to the elongation 9°. and angle A equal to ATB or to the length of the tail, *viz.* 35°. then SA will be to SΓ, that is, the line of the greateſt poſſible diſtance of the Comet from the Sun, to the ſemidiameter of the orbis magnus, as the ſine of the angle T to the ſine of the angle A, that is, as about 3 to 11. And therefore the Comet at that time was leſs diſtant from the Sun than by $\frac{3}{11}$ of the Earth's diſtance from the ſame, and conſequently either was within the orbit of Mercury, or between that orbit and the Earth. Again, *Dec.* 21. the elongation of the Comet from the Sun was $32\frac{2}{5}$°, and the length of its tail 70°. Wherefore as the ſine of $32\frac{2}{5}$° to the ſine of 70°, that is, as 4 to 7, ſo was the limit of the Comet's diſtance from the Sun to the diſtance of the Earth from the Sun, and conſequently the Comet had not then got without the orbit of

Venus. *Dec.* 28. the elongation of the Comet from the Sun was 55°. and the length of its tail 56°. And therefore the limit of the Comet's diſtance from the Sun was not yet equal to the diſtance of the Earth from the ſame, and conſequently the Comet had not then got without the Earth's orbit. But from its parallax we find that its egreſs from the orbit happened about *Jan.* 5. as well as that it had deſcended far within the orbit of Mercury. Let us ſuppoſe it to have been in its perihelion *Dec.* the 8th when it was in conjunction with the Sun, and it will follow that in the journey from its perihelion to its exit out of the Earth's orbit, it had ſpent 28 days; and conſequently that in the 26 or 27 days following, in which it ceaſed to be further ſeen by the naked eye, it had ſcarce doubled its diſtance from the Sun. And by limiting the diſtances of other Comets by the like arguments we come at laſt to this concluſion, That all Comets during the time in which they are viſible by us, are within the compaſs of a ſpherical ſpace deſcribed about the Sun as a center, with a radius double, or at moſt triple of the diſtance of the Earth from the Sun.

AND hence it follows that the Comets, during the whole time of their appearance unto us, being within the ſphere of activity of the circum-ſolar force, and therefore agitated by the impulſe of that force, will (for the ſame reaſon as the Planets) be made to move in conic ſections that have one focus in the center of the Sun, and, by radii drawn to the Sun, to deſcribe area's proportional to the times. For that force is propagated to an immenſe diſtance, and will govern the motions of bodies far beyond the orbit of Saturn.

That the Comets move in conic-ſections, having one focus in the center of the Sun, and by radii drawn to that center do deſcribe area's proportional to the times.

THERE are three hypotheſes about Comets. For ſome will have it that they are generated and periſh, as often as they appear and vaniſh; others that they come from the regions of the fixed Stars, and are ſeen by us in their paſſage through the ſyſtem of our Planets; and laſtly, others that they are bodies perpetually revolving about the Sun in very eccentric orbits. In the firſt caſe, the Comets, according to their different velocities, will move in conic ſections of all ſorts; in the ſecond they

That thoſe conic-ſections are near to parabola's, proved from the velocity of the Comets.

will describe hyperbola's, and in either of the two will frequent indifferently all quarters of the heavens as well those about the poles as those towards the Ecliptic; in the third their motions will be performed in ellipses very eccentric and very nearly approaching to parabola's. But (if the law of the Planets is observed) their orbits will not much decline from the plain of the Ecliptic. And so far as I could hitherto observe the third case obtains. For the Comets do indeed chiefly frequent the zodiac and scarce ever attain to a heliocentric latitude of 40°. And that they move in orbits very nearly parabolical, I infer from their velocity. For the velocity with which a parabola is described, is every where to the velocity with which a Comet or Planet may be revolved about the Sun in a circle at the same distance, in the subduplicate ratio of 2 to 1. And by my computation the velocity of Comets is found to be much about the same. I examined the thing by inferring nearly the velocities from the distances, and the distances both from the parallaxes and the phænomena of the tails, and never found the errours of excess or defect in the velocities greater than what might have arose from

the

the errours in the diſtances collected after that manner. But I likewiſe made uſe of the reaſoning that follows.

In what ſpace of time Comets deſcribing parabolic trajectories paſs through the ſphere of the orbis magnus.

SUPPOSING the radius of the orbis magnus to be divided into 1000 parts: Let the numbers in the firſt column of the following table repreſent the diſtance of the vertex of the parabola from the Sun's center, expreſſed by thoſe parts; and a Comet in the times expreſſed in Col 2. will paſs from its perihelion to the ſurface of the ſphere which is deſcribed about the Sun as a center with the radius of the orbis magnus; and in the times expreſſed in Col. 3, 4, and 5. it will double, triple, and quadruple that its diſtance from the Sun.

TABLE I.

The distance of a Comet's perihelion from the Sun's center.	The time of a Comet's passage from its perihelion to a distance from the Sun equal to the rad. of the orbis mag.	To its double.	To its triple.	To its quadruple.
	d. h. '.	d. h. '.	d. h. '.	d. h. '.
0	27 11 12	77 16 28	142 17 14	219 17 30
5	27 16 07	77 23 14		
10	27 21 00	78 06 24		
20	28 06 40	78 20 13	144 03 19	221 08 54
40	29 01 32	79 23 33		
80	30 13 26	82 04 56		
160	33 05 29	86 10 26	153 16 08	232 12 19
320	37 13 46	93 23 38		
640	37 09 49	105 01 28		
1280		106 06 35	200 06 43	297 03 46
2560			147 22 31	300 06 03

[This table, here corrected, is made on the supposition that the Earth's diurnal motion is just 59'. and the measure of one minute loosely 0,2909, in respect of the radius 1000. If those measures are taken true, the true numbers of the table will all come out less. But the difference, even when greatest, and to the quadruple of the Earth's distance from the Sun, amounts only to 16 h. 55'.]

THE

At what time Comets enter into and paſs out of the ſphere of the orbis magnus.

THE time of a Comet's ingreſs into the ſphere of the orbis magnus, or of its egreſs from the ſame may be inferred nearly from its parallax, but with more expedition by the following

TABLE II.

The apparent elongation of a Comet from the Sun.	Its apparent diurnal motion in its own orbit.		Its diſtance from the Earth in parts, whereof the radius of the orbis magnus contains 1000.
	Direct	Retrog.	
60°	2° 18′	00° 20′	1000
65	2 38	00 35	845
70	2 55	00 57	684
72	3 07	01 09	618
74	3 23	01 29	551
76	3 43	01 45	484
78	4 10	02 12	416
80	4 47	02 49	347
82	5 45	03 47	278
84	7 18	05 20	209
86	10 27	08 19	140
88	18 37	16 39	70
90	Infinite	Infinite	00

With what velocity the Comets of 1680 paſſed through the ſphere of the orbis magnus.

THE ingreſs of a Comet into the ſphere of the orbis magnus, or its egreſs from the ſame happens at the time of its elongation from the Sun expreſſed in Col. I. againſt its diurnal motion.

So in the Comet of 1681, *Jan.* 4. *O. S.* the apparent diurnal motion in its orbit was about 3°. 05′. and the correspond-ing elongation 71 ⅔°. And the Comet had acquired this elongation from the Sun *Jan.* 4. about six in the morning. Again in the year 1680, *November* 11. the diurnal motion of the Comet that then appeared, was about 4 ⅔°. and the corresponding elongation 79 ⅔ happened *Nov.* 10. a little before mid-night. Now at the times named these Comets had arrived at an equal distance from the Sun with the Earth, and the Earth was then almost in its perihelion. But the first table is fitted to the Earth's mean distance from the Sun assumed of 1000 parts; and this distance is greater by such an excess of space as the Earth might describe, by its annual motion, in one days time, or the Comet, by its motion, in 16 hours. To reduce the Comet to this mean distance of 1000 parts we add those 16 hours to the former time and subduct them from the latter; and thus the former becomes *Jan.* 4^{d}. 10^{h}. after noon, the latter *Nov.* 10. about six in the morning. But from the tenour and progress of the diurnal motions it appears that both Comets were in conjunction with the Sun

between

between *Dec*. 7. and *Dec*. 8. And from thence to *Jan*. 4^d. 10^h. afternoon on one side, and to *Nov*. 10^d. 6^h. of the morning on the other, there are about 28 days. And so many days (by *Table* 1.) the motions in parabolic trajectories do require.

That these were not two, but one and the same Comet. In what orbit and with what velocity this Comet was carried through the heavens described more exactly.

But though we have hitherto considered those Comets as two, yet from the coincidence of their perihelions and agreement of their velocities, it is probable, that in effect they were but one and the same. And if so, the orbit of this Comet must have either been a parabola or at least a conic section very little differing from a parabola, and at its vertex almost in contact with the surface of the Sun. For by *Tab*. 2. the distance of the Comet from the Earth *Nov*. 10. was about 360 parts, and *Jan*. 4. about 630. From which distances, together with its longitudes and latitudes, we infer the distance of the places in which the Comet was at those times, to have been about 280: the half of which, *viz*. 140 is an ordinate to the Comet's orbit, cutting off a portion of its axe nearly equal to the radius of the orbis magnus, that is to

1000

1000 parts. And therefore dividing the square of the ordinate 140 by 1000 the segment of the axe, we find the latus rectum 19, 16, or in a round number 20. The fourth part whereof 5 is the distance of the vertex of the orbit from the Sun's center. But the time corresponding to the distance of 5 parts in *Tab.* 1. is $27^d. 16^h. 7'$. In which time, if the Comet moved in a parabolic orbit, it would have been carried from its perihelion to the surface of the sphere of the orbis magnus described with the radius 1000, and would have spent the double of that time, *viz.* $55^d. 8\frac{1}{4}^h$. in the whole course of its motion within that sphere: And so in fact it did. For from *Nov.* $10^d. 6^h$. of the morning, the time of the Comet's ingress into the sphere of the orbis magnus, to *Jan.* $4^d. 10^h$. after noon, the time of its egress from the same, there are $55^d. 16^h$. The small difference of $7\frac{3}{4}^h$. in this rude way of computing is to be neglected, and perhaps may arise from the Comet's motion being some small matter slower, as it must have been if the true orbit in which it was carried was an ellipse. The middle time between its ingress and egress was *December* $8^d. 02^h$. of the morning. And there-

therefore at this time the comet ought to have been in its perihelion. And accordingly that very day, juſt before Sun-riſing, Dr. *Halley* (as we ſaid) ſaw the tail ſhort and broad, but very bright riſing perpendicularly from the horizon. From the poſition of the tail, it is certain that the Comet had then croſſed over the Ecliptic, and got into north-latitude, and therefore had paſſed by its perihelion which lay on the other ſide of the Ecliptic, though it had not yet come into conjunction with the Sun. And the Comet, being at this time between its perihelion and its conjunction with the Sun, muſt have been in its perihelion a few hours before. For in ſo near a diſtance from the Sun it muſt have been carried with great velocity, and have apparently deſcribed almoſt half a degree every hour.

With what velocity Comets are carried, ſhewed by more examples.

By like computations I find that the Comet of 1618 entered the ſphere of the orbis magnus *December* 7. towards Sun-ſetting. But its conjunction with the Sun was *Nov.* 9, or 10, about 28 days intervening, as in the preceding Comet. For from the ſize of the tail of this, in which it was equal to the preceding, it

is

is probable that this Comet likewise did come almost into a contact with the Sun. Four Comets were seen that year, of which this was the last. The second, which made its first appearance *October* 31. in the neighbourhood of the rising Sun, and was soon after hid under the Sun's rays, I suspect to have been the same with the fourth, which emerged out of the Sun's rays about *Nov.* 9. To these we may add the Comet of 1607, which entered the sphere of the orbis magnus *Sept.* 14. *O. S.* and arrived at its perihelion-distance from the Sun about *October* 19, 35 days intervening. Its perihelion-distance subtended an apparent angle at the Earth of about 23 degrees, and was therefore of 390 parts. And to this number of parts about 34 days correspond in *Tab.* 1. Further, the Comet of 1665 entered the sphere of the orbis magnus about *March* 17, and came to its perihelion about *April* 16. 30 days intervening. Its perihelion-distance subtended an angle at the Earth of about seven degrees, and therefore was of 122 parts: and corresponding to this number of parts, in *Tab.* 1. we find 30 days. Again, the Comet of 1682 entered the sphere of the orbis magnus about *Aug.* 11. and arrived at its

its perihelion about *Sept.* 16. being then diſtant from the Sun by about 350 parts, to which, in *Tab.* 1. belong 33½ days. Laſtly, that memorable Comet of *Regiomontanus*, which in 1472 was carried through the circum-polar parts of our northern hemiſphere with ſuch rapidity as to deſcribe 40 degrees in one day, entered the ſphere of the orbis magnus *Jan.* 21. about the time that it was paſſing by the pole, and haſtening from thence towards the Sun, was hid under the Sun's rays about the end of *February*. Whence 'tis probable that 30 days or a few more were ſpent between its ingreſs into the ſphere of the orbis magnus and its perihelion. Nor did this Comet truly move with more velocity than other Comets, but owed the greatneſs of its apparent velocity to its paſſing by the Earth at a near diſtance.

The inveſtigation of the trajectory of Comets propoſed.

It appears then that the velocity of Comets, ſo far as it can be determined by theſe rude ways of computing, is that very velocity with which parabola's, or ellipſes near to parabola's, ought to be deſcribed. And therefore the diſtance between a Comet and the Sun being given,

given, the velocity of the Comet is nearly given. And hence ariſes this problem.

The relation betwixt the velocity of a Comet, and its diſtance from the Sun's center being given, the Comets trajectory is required.

If this problem was reſolved, we ſhould thence have a method of determining the trajectories of Comets to the greateſt accuracy. For if that relation be twice aſſumed, and from thence the trajectory be twice computed, and the errour of each trajectory be found from obſervations, the aſſumption may be corrected by the Rule of Falſe, and a third trajectory may thence be found that will exactly agree with the obſervations. And by determining the trajectories of Comets after this method, we may come at laſt to a more exact knowledge of the parts through which thoſe bodies travel, of the velocities with which they are carried, what ſort of trajectories they deſcribe, and what are the true magnitudes and forms of their tails according to the various diſtances of their heads from the Sun; whether after certain intervals of time,

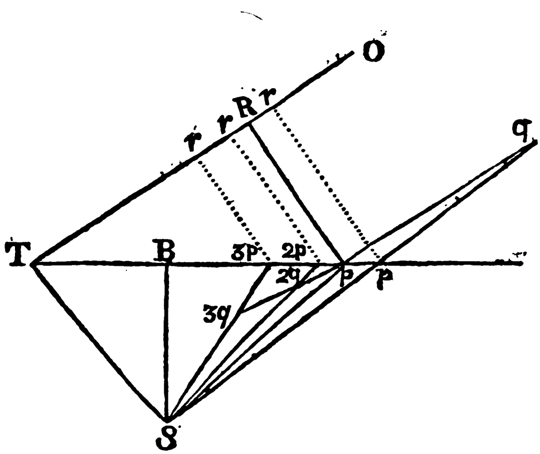
the ſame Comets do return again, and in what periods they compleat their ſeveral revolutions. But the problem may be reſolved by determining firſt the hourly motion of a Comet to a given time from three or more obſervations, and then deriving the trajectory from this motion. And thus the invention of the trajectory depending on one obſervation and its hourly motion at the time of this obſervation will either confirm or diſprove it ſelf. For the concluſion that is drawn from the motion only of an hour or two and a falſe hypotheſis, will never agree with the motions of the Comets from beginning to end. The method of the whole computation is this.

Lemma's premiſed to the ſolution of the problem.

Lem. 1. To cut two right lines OR, TP given in poſition, by a third right line RP, ſo as TRP may be a right angle, and, if another right line SP is drawn to any given point S, the ſolid contained under this line SP, and the ſquare of the right line OR terminated at a given point O, may be of a given magnitude.

It is done by linear deſcription, thus. Let the given magnitude of the ſolid be $M^2 \times N$. From any point *r* of the right line

line OR erect the perpendicular rp meeting TP in p. Then through the points Sp draw the line Sq equal to $\frac{M^2 \times N}{Or^2}$. In like manner draw three or more right lines $S2q$, $S3q$, &c. And a regular line $q2q3q$ drawn through all the points $q2q3q$, &c. will cut the right line TP in the point P, from which the perpendicular PR is to be let fall. Q. E. F.



By trigonometry thus. Assuming the right line TP as found by the preceding method, the perpendiculars TR, SB in the triangles TPR, TPS will be thence given, and the side SP in the triangle SBP

SBP, as well as the errour $\frac{M^2 \times N}{OR^2} - SP$.

Let this errour, suppose D, be to a new errour, suppose E, as the errour $2p\,2q \pm 3p\,3q$ to the errour $2p\,3p$; or as the errour $2p\,2q \pm D$ to the errour $2p\,P$; and this new errour added to or subducted from the length TP will give the correct length $TP \pm E$. The inspection of the figure will shew whether we are to add or to subtract. And if at any time there should be use for a further correction, the operation may be repeated.

By arithmetic thus. Suppose the thing done, and let $TP \pm e$ be the correct length of the line TP found by delineation; and the correct lengths of the lines OR, BP and SP will be $OR \pm \frac{TR}{TP}e$, $BP \pm e$ and $\sqrt{SP^2 \pm 2BPe \pm e^2}$ $= \frac{M^2 \times N}{OR \pm \frac{2OR \times TR}{TP}e + \frac{TR^2}{TP^2}e^2}$.

The sign $\pm$ denotes $+$ or $-$ ambiguously, and $\mp$ stands for the contrary sign.

Whence by the method of converging series's, we have $SP \pm \frac{BP}{SP}e + \frac{SB^2}{2SP^3}ee$, &c. $= \frac{M^2 N}{OR^2} \mp \frac{M^2 N}{OR^3}e + \frac{M^2 N}{OR^4}e^2$, &c.

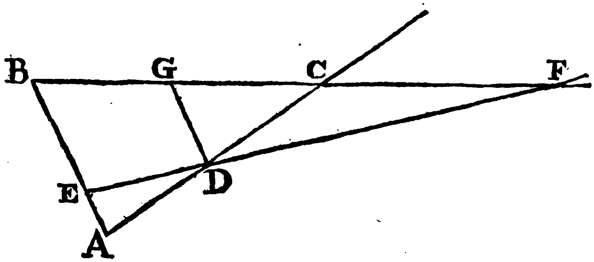
And for the given co-efficients $\frac{M^2 N}{OR^2}$ $-SP, \pm \frac{BP}{SP} \pm \frac{M^2 N}{OR^3}, \frac{M^2 N}{OR^4} - \frac{SB^2}{2SP^3}$ putting $F, \frac{F}{G}, \frac{F}{GH}$, and carefully observing the signs, we find $GH - e^2 = He$, or $G - \frac{e^2}{H} = e$. Whence, neglecting the very small term $\frac{e^2}{H}$, e comes out equal to G. If the errour $\frac{e^2}{H}$ is not despicable, take $e = G - \frac{G^2}{H}$.

And it is to be observed, that here a general method is hinted at for solving the more intricate sort of problems, as well by trigonometry as by arithmetic, without those perplexed computations and resolutions of affected equations, which hitherto have been in use.

Lem. 2. To cut three right lines given in position by a fourth right line that shall pass through a point assigned in any of the three, and so as its intercepted parts shall be in a given ratio one to the other.

Let

Let A B, A C, B C be the right lines given in poſition, and ſuppoſe D to be the given point in the line AC. Parallel to AB draw DG meeting BC in G. And,



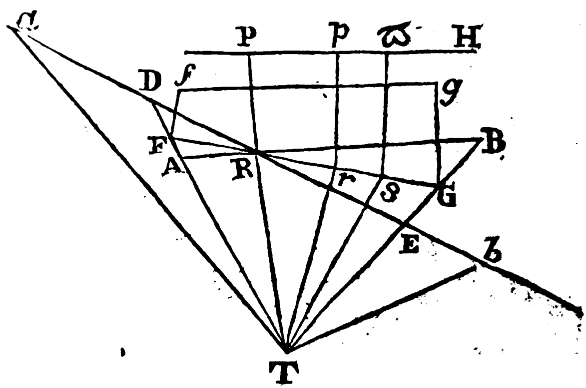
taking GF to BG in the given ratio, draw F D E; and FD will be to DE as FG to BG. Q. E. F.

By trigonometry thus. In the triangle CGD, all the angles and the ſide CD are given, and from thence its remaining ſides are found; and from the given ratio's, the lines GF and B E are alſo given.

Lem. 3. To find and repreſent by a linear deſcription the hourly motion of a Comet to any given time.

From obſervations of the beſt credit, let three longitudes of the Comet be given, and ſuppoſing ATR, RTB to be their differences; let the hourly motion be required to the time of the middle obſer-

obſervation TR. By *Lem.* 2. draw the right line ARB, ſo as its intercepted parts AR, RB may be as the times between the obſervations. And if we ſuppoſe a body in the whole time to deſcribe the whole line AB with an e-



qual motion, and to be in the mean time viewed from the place T, the apparent motion of that body about the point R, will be nearly the ſame with that of the Comet at the time of the obſervation TR.

The ſame more accurately.

Let T*a*, T*b* be two longitudes given at a greater diſtance on one ſide and on the other; and by *Lem.* 2. draw the right line *a*R*b* ſo as its intercepted parts *a*R, R*b* may be as the times between the

the obſervations *a* T R, R T *b*. Suppoſe this to cut the lines TA, TB in D and E. And becauſe the errour of the inclination T R *a* increaſes nearly in the duplicate ratio of the time between the obſervations, draw FRG, ſo as either the angle D R F may be to the angle A R F, or the line D F to the line AF, in the duplicate ratio of the whole time between the obſervations *a* T B to the whole time between the obſervations AT B, and uſe the line thus found F G in place of the line AB found above.

It will be convenient that the angles ATR, RTB, *a* T A, BT *b* be no leſs than of ten or fifteen degrees, the times correſponding no greater than of eight or twelve days, and the longitudes taken when the Comet moves with the greateſt velocity. For thus the errours of the obſervations will bear a leſs proportion to the differences of the longitudes.

Lem. 4. To find the longitudes of a Comet to any given times.

It is done by taking, in the line F G, the diſtances R *r*, R ρ proportional to the times, and drawing the lines T *r*, T ρ. The way of working by trigonometry is manifeſt.

Lem.

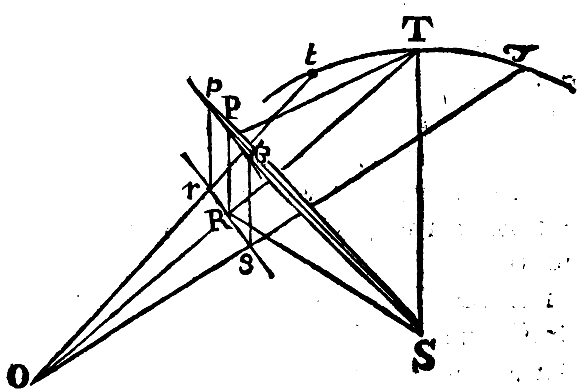
Lem. 5. To find the latitudes.

On TF, TR, TG, as radius's, at right angles erect F*f*, R P, G*g* tangents of the obſerved latitudes ; and parallel to *fg* draw PH. The perpendiculars *rp*, *ρϖ* meeting P H will be the tangents of the ſought latitudes to T*r* and T*ρ* as radius's.

The problem reſolved.

Prob. 1. From the aſſumed ratio of the velocity to determine the trajectory of a Comet.

Let S repreſent the Sun, *t*, T, *τ* three places of the Earth in its orbit at equal



diſtances, *p*, P, *ϖ* as many correſponding places of the Comet in its trajectory, ſo as the diſtances interpoſed betwixt place and place may anſwer to the motion of one hour, *pr*, PR, *ϖρ* perpendiculars let

let fall on the plain of the ecliptic, and $r\,R\,\rho$ the vestige of the trajectory in this plain. Join $S\,p$, SP, $S\,\varpi$, SR, ST, tr, TR, $\tau\rho$, TP; and let $t\,r$, $\tau\rho$ meet in O, TR will nearly converge to the same point O, or the errour will be inconsiderable. By the premised lemma's, the angles rOR, ROρ are given, as well as the ratio's $p\,r$ to $t\,r$, PR to TR, and $\varpi\rho$ to $\tau\rho$. The figure $t\,T\,\tau\,O$ is likewise given, both in magnitude and position, together with the distance ST and the angles STR, PTR, STP. Let us assume the velocity of the Comet, in the place P, to be to the velocity of a Planet, revolved about the Sun in a circle, at the same distance SP, as V to 1, and we shall have a line $p\,P\,\varpi$ to be determined, of this condition, that the space $p\,\varpi$, described by the Comet in two hours, may be to the space $V \times t\,\tau$ (that is, to the space which the Earth describes in the same time multiplied by the number V) in the subduplicate ratio of ST, the distance of the Earth from the Sun, to SP the distance of the Comet from the Sun; and that the space pP, described by the Comet in the first hour, may be to the space $P\varpi$ described by the Comet in the second hour, as the velocity in p, to the velocity in P,

2 that

that is, in the subduplicate ratio of the distance SP to the distance Sp, or in the ratio of 2 SP, to SP+Sp. For in this whole work I neglect small fractions that can produce no sensible errour.

In the first place then, as mathematicians in the resolution of affected equations, are wont, for the first essay, to assume the root by conjecture; so, in this analytical operation, I judge of the sought distance TR, as I best can by conjecture. Then by *Lem.* 2. I draw $r\rho$, first supposing rR equal to Rρ, and again (after the ratio of SP to Sp is discovered) so as rR may be to Rρ, as 2 SP to SP+Sp, and I find the ratio's of the lines $p\varpi$, $r\rho$ and OR one to the other. Let M be to V$\times t\tau$ as OR to $p\varpi$; and because the square of $p\varpi$ is to the square of V$\times t\tau$, as ST to SP, we shall have *ex æquo* OR^2 to M^2 as ST to SP, and therefore the solid OR^2 $\times$SP equal to the given solid M$^2\times$ST. Whence (supposing the triangles STP, PTR to be now placed in the same plain) TR, TP, SP, PR will be given by *Lem.* 1. All this I do, first by delineation in a rude and hasty way, then by a new delineation with greater care, and lastly, by an arithmetical computa-

tion.

tion. Then I proceed to determine the poſition of the lines $r\rho$, $p\varpi$ with the greateſt accuracy, together with the nodes and inclination of the plain $Sp\varpi$ to the plain of the ecliptic; and in that plain $Sp\varpi$, I deſcribe the trajectory in which a body let go from the place P in the direction of the right line $p\varpi$, would be carried with a velocity that is to the velocity of the Earth, as $p\varpi$ to $V \times t\tau$. Q. E. F.

Prob. 2. To correct the aſſumed ratio of the velocity and the trajectory thence found.

Take an obſervation of the Comet about the end of its appearance, or any other obſervation at a very great diſtace from the obſervations uſed before, and find the interſection of a right line drawn to the Comet, in that obſervation, with the plain $Sp\varpi$, as well as the Comet's place in its trajectory to the time of the obſervation. If that interſection happens in this place, it is a proof that the trajectory was right determined. If otherwiſe, a new number V is to be aſſumed, and a new trajectory to be found, and then the place of the Comet in this trajectory to the time of that probatory obſervation, and the interſection of a right line drawn

to the Comet, with the plain of the trajectory are to be determined as before. And by comparing the variation of the errour with the variation of the other quantities, we may conclude, by the Rule of Three, how far those other quantities ought to be varied or corrected, so as the errour may become as small as possible. And by means of these corrections we may have the trajectory exactly, providing the observations, upon which the computation was founded, were exact, and that we did not err much in the assumption of the quantity V; for if we did, the operation is to be repeated till the trajectory is exactly enough determined. Q. E. F.

FINIS.

ERRATA.

PREFACE, page 15. line 4. from the bottom, dele *two*. Pag. 23. line 19. read *as the sum* &c. Pag. 43. lin. 6. for *and* read *then*. Pag. 53. lin. 17. between 520116 and 72333 insert 152399. In the Scheme pag. 84. draw a line from *b* through C to A. Pag. 119. lin. 7. read *about* ♑ 18°. 26′. Pag. 134. the first number in col. 5. should have stood even with those in the other four.